Original Article

Changes in Pre-service Teachers' Noticing Through an Elementary Mathematics Methods Course Combining Theory and Practice

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Abstract

The purpose of this study was to analyze the changes in pre-service teachers' noticing through an elementary mathematics methods course along with a practicum. For this purpose, the comments pre-service teachers made on an entire video-based mathematics lesson were collected three times over the semester. Their comments were analyzed in terms of topic, actor, stance, evidence, and alternative strategy. The results showed that the pre-service teachers' noticing abilities were slightly changed after learning mathematical content and pedagogy related to teaching elementary mathematics. Substantial changes in their noticing occurred after a four-week practicum and subsequent discussions on their own lesson planning, implementation, and reflections. This study has implications for designing a mathematics methods course to develop teacher expertise and refining a teacher noticing analytic framework.

Keywords: pre-service teachers, teacher noticing, elementary mathematics methods course, teacher preparation program

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I. INTRODUCTION

Given the significant role that a teacher plays in students' meaningful mathematics learning, pre-service teachers [PSTs] must develop a variety of classroom expertise through teacher preparation programs. Some studies have explored what kinds of knowledge, beliefs, and identities related to mathematics and mathematics teaching are needed, specifically for teachers (Potari & Chapman, 2020). Others have focused on how to foster PSTs' actual instructional practices using various tools and processes, such as lesson videos, curriculum resources, technology, or debrief conversations for mathematics lessons (Llinares & Chapman, 2020; van Es & Sherin, 2008). Over the past few decades, teacher educators have made an effort to provide PSTs with more opportunities to develop their knowledge and orientation of mathematics teaching, and to foster actual teaching practice through a teacher preparation program.

Despite the common approach of emphasizing both theoretical and practical aspects in a teacher preparation program, research is lacking on how effective such programs are for enhancing teacher expertise. Among many other indicators, a teacher's ability to notice what is mathematically significant in a complex classroom situation has been identified as a key indicator of teacher expertise (Mason, 2002; Mitchell & Marin, 2015; van Es & Sherin, 2008). This is mainly because teachers' instructional decision-making is based on their identification of and reasoning about the salient features of students' mathematics learning (Jacobs, Lamb, & Philipp, 2010; Mason, 2002). In fact, according to the Association of Mathematics Teacher Educators [AMTE] (2017), well-prepared beginning mathematics teachers are required to "attend to students' thinking about mathematics content" and "recognize students' engagement in mathematical practices" (pp. 18-19). In this regard, *teacher noticing*, the ability to identify and interpret particular events in an instructional context has been emphasized as a core competence of mathematics teachers (Jacobs et al., 2010; Sherin, Jacobs, & Philipp, 2011).

In order to develop the ability to notice in a teacher preparation program, teacher educators often provide PSTs with specific pedagogical approaches in which they have opportunities to observe and analyze others' videos (e.g., Ulusoy, 2020; van Es et al., 2017) or their own lessons (e.g., Roller, 2016; Santagata & Yeh, 2014). These approaches have been reported as effective in developing teacher noticing. However, studies usually do not analyze how PSTs develop their noticing skills over time via the knowledge and experience they gain in a teacher preparation program.

This paper analyzes the *changes* of PSTs' noticing achieved through an elementary mathematics methods course in the Korean context. While taking the university course for one semester, the PSTs first learned the theory of teaching elementary mathematics, such as content knowledge and pedagogical knowledge. Next, they had a four-week practicum period in preassigned nearby elementary schools. Finally, the PSTs came back to the university and had debrief discussions about their enacted mathematics lessons. By measuring their noticing ability three times during the semester,

this study examined the changes in the PSTs' noticing in the teacher preparation program context.

The research questions were as follows: (a) How does PSTs' noticing in a mathematics lesson change after learning theoretical aspects during a university course compared to their noticing at the beginning of the course? (b) How does PSTs' noticing in a mathematics lesson change after discussing their own mathematics lessons implemented during the practicum period? Two common pedagogical approaches for educating PSTs on how to teach elementary mathematics are the following: increase the PSTs' knowledge of mathematics and mathematics teaching and allow the PSTs to implement lessons in an elementary classroom, with subsequent discussion. This paper analyzes what provokes such changes in teacher noticing and describes implications for designing a mathematics methods course.

II. LITERATURE REVIEW

1. Development of noticing skills through teacher education programs

Expert teachers can identify and understand mathematically meaningful situations in a lesson and make appropriate decisions that foster students' mathematics learning (Jacobs et al., 2010; Mason, 2002; van Es & Sherin, 2008). For instance, expert teachers can attend to students' mathematical thinking and provide interpretive comments, whereas novice teachers attend to the whole class environment and provide primarily descriptive comments (van Es, 2011). In a similar vein, expert teachers can present robust evidence in interpreting students' mathematical thinking, but novice teachers provide limited or no evidence (Jacobs et al., 2010). Given the premise that noticing is a learnable practice (Jacobs & Spangler, 2017), considerable attempts have been made to support the development of PSTs' noticing skills through teacher education programs (Schack, Fisher, & Wilhelm, 2017).

To develop noticing skills, PSTs need to have knowledge (or resources) to make sense of what is identified as noteworthy features of classroom interactions. In this regard, Sherin and van Es (2009) emphasized *knowledge-based reasoning* and van Es (2011) proposed that only teachers with high-level noticing skills can "make connections between events and principles of teaching and learning" (p. 139). In fact, recent studies of teacher noticing have emphasized the correlation between teacher knowledge on specific mathematical topics and noticing ability in interpreting students' learning. For instance, Dreher and Kuntze (2015) argued that specific content knowledge, such as knowing how to explain the process of calculating fractions in different ways or how to connect different expressions, is more important for teacher noticing than general content knowledge, such as knowing how to calculate fractions. However, a typical elementary mathematics methods course, which is the main course for PSTs to learn how to teach elementary mathematics, struggles to cover all details of specific content knowledge within one semester. Little is known about how learning to teach elementary mathematics through a general knowledge-driven

methods course impacts PSTs' noticing skills.

Many studies on enhancing PSTs' noticing skills have involved lesson videos (Mitchell & Marin, 2015; Santagata & Yeh, 2014; van Es et al, 2017) with which they highlight specific events or moments of mathematical thinking and interpret them. Such videos were selected mostly by the researchers, tended to be short (within 10 minutes), and featured students' particular strategies or thinking processes. In addition to using videos, recent studies have emphasized more specific noticing based on the structured framework for analysis. Such studies have provided PSTs with specific prompts, stimulating them to pay closer attention to the important aspects of a mathematics lesson. For example, McDuffie et al. (2014) provided a specific prompt called *Four Lenses* (i.e., teaching, learning, tasks, and power and participation lenses) for PSTs to use when observing and discussing video clips selected deliberately to represent students' mathematical thinking. In their video club with four PSTs, Mitchell and Marin (2015) employed a protocol focusing on the *Mathematical Quality of Instruction* defined by Hill et al. (2008). Using the protocol, the teacher educators helped PSTs concentrate on the key components of high-quality mathematics lessons, such as eliciting student explanations.

These studies identified the characteristics of PSTs' noticing and further explored how to foster such a noticing ability through a university-based course. However, the studies were conducted in deliberately designed contexts or interventions to enhance PSTs' noticing ability. For example, they used short video excerpts, like those already mentioned, discussions of salient interactions within the videos, and specific prompts or protocols to guide noticing. It is plausible that the targeted interventions lead to the successful development of PSTs' noticing skills (van Es et al, 2017). However, observing a salient part of an entire lesson is different from understanding the overall situation of the lesson. Given that a mathematics lesson involves complex situations in which multiple factors are intertwined (Lampert, 2001), PSTs need to be able to observe the lesson as a whole to fully understand the interactions among the teacher, students, and content (Pang, 2011). This study examined how and what aspects PSTs pay attention to in an elementary mathematics lesson as a whole.

Another aspect that has been overlooked in previous studies is the influence of the PSTs' practicum and its related learning opportunities on their noticing skills. During a practicum period, PSTs have an opportunity to apply what they have learned through a university course to actual practices in mathematics lessons. They can analyze and reflect on their own lessons instead of others' videos. Taken together, we as education researchers need to explore the nature and quality of the noticing ability emergent in a rather naturalistic teacher education program which consists of both university lecture and practicum.

Some studies have reported positive or improved outcomes of PSTs' noticing through a teacher preparation program's intervention (e.g., Mitchell & Marin, 2015; Star & Strickland, 2008; van Es et al., 2017). But others have reported declines in PSTs' noticing (e.g., Castro Superfine et al., 2015). In fact, after reviewing 27 studies on PSTs' noticing, Amador, Bragelman, and Castro Superfine (2021) concluded that

a majority of the studies showed “stagnant, mixed level, or inconsistent results” (p. 11). The researchers also highlighted the divergent and ambiguous data collection and analytic methods used to study PSTs’ noticing. These findings imply that researchers on PSTs’ noticing need to be clear about what theoretical framework is used and what methodological approaches are taken, which are elaborated upon in the next section.

2. Teacher noticing framework

Teacher noticing is mostly analyzed in terms of what to notice and how to notice (Amador et al., 2021). This study is based on the Framework for Learning to Notice Student Mathematical Thinking by van Es (2011). The framework classifies four levels of noticing: Level 1 (Baseline), Level 2 (Mixed), Level 3 (Focused), and Level 4 (Extended). For instance, teachers at Level 1 attend to the whole class environment, behavior, and teacher pedagogy, providing descriptive and evaluative comments with little evidence to support their analysis. In contrast, teachers at Level 4 attend to the relationship between particular students’ mathematical thinking and teaching strategies, providing interpretive comments by referring to specific events and interactions as evidence. This study did not employ the levels of noticing per se but used the analytic elements in the framework to present detailed analyses of what and how PSTs notice.

Firstly, regarding what to notice, both *actor* (i.e., the individual[s] PSTs notice) and *topic* (i.e., the issue PSTs identify) are the main analytic elements (van Es, 2011; van Es & Sherin, 2008). Actors are generally teachers or students, even though this term can be further classified as the classroom as a whole, students, an individual student, the teacher in the video clip, or teachers themselves (Stockero, Rupnow, & Pascoe, 2017; van Es & Sherin, 2008). Teachers at higher levels of noticing tend to identify particular students’ mathematical thinking, but teachers at lower levels tend to focus on teacher behavior or pedagogy (van Es, 2011). In a similar vein, PSTs’ student-focused comments are regarded as higher-level performance than teacher-focused comments when evaluating teachers’ ability to analyze a mathematics lesson (Santagata & Yeh, 2014). It is fundamental to analyze who PSTs notice to trace changes in their noticing skills. The *actor* element may be more important when a whole mathematics lesson is studied in place of a short video clip because the latter is often selected for its noteworthy aspects of particular students’ mathematical thinking or activities (Stockero et al., 2017).

Topics are subtly different from study to study. For instance, Star and Strickland (2008) classified the topics that PSTs noticed from an entire video: the classroom environment, classroom management, mathematical content, tasks, and communication. In another case, van Es et al. (2017) classified the topics that PSTs noticed from a short video clip, specifying the mathematical content and learning goals, student thinking, pedagogies for making thinking visible, and classroom discourse norms. These studies analyzed the frequency of such topics and showed which topics were most often noticed by PSTs. However, such studies were not specific regarding what would be noticed in detail per topic.

Our study expanded the trend of previous studies by further specifying each topic. For instance, the topic of Mathematical Tasks can be further classified depending on whether the main focus is to align with the lesson objective, to adjust students' interest or motivation to learn, or to promote students' mathematical thinking. Specifying each topic can reveal more precisely not only what PSTs notice but also what they fail to or struggle to notice (Spizer & Phelps-Gregory, 2017). As this study analyzes changes in PSTs' noticing through a mathematics methods course for one semester, knowing exactly what PSTs notice and what they miss can help teacher educators better design a methods course.

Secondly, how to notice is related to the ways teachers describe and interpret what they notice. Noticing ability requires teachers to interpret and reason about what they notice. In this sense, van Es and Sherin (2008) analyzed how the teachers participating in a video club made inferences about the classroom events they observed on the basis of their *stance*. Specifically, the researchers analyzed whether the teachers simply described the events that occurred in the video clip, evaluated them as being good or bad, or interpreted them. These different stances align with the depth of interpretation of teachers' responses or opinions in classroom video analysis (Santagata & Sandholtz, 2019).

Teachers are also expected to provide reasonable *evidence* regarding their descriptions or interpretations of classroom events. In this regard, teacher noticing is often evaluated on whether teachers referred to specific events and interactions as evidence (van Es, 2011) or whether they interpreted students' mathematical thinking based on robust evidence (Jacobs et al, 2010). These schemes have primarily focused on the existence and conviction of evidence to support analysis. However, the sources of evidence are just as important because teacher noticing is related to knowledge, orientation, or the experience of mathematics teaching and learning (Blömeke, Kaiser, König, & Jentsch, 2020).

Another aspect regarding how to notice is whether teachers can present a detailed description of an alternative teacher action or justification for using a different instructional move. For instance, Santagata, Zannoni, and Stigler (2007) used the criteria of *alternative strategies* in analyzing teacher comments. Teachers at the highest level of noticing are expected to propose alternative pedagogical solutions (van Es, 2011). In a similar vein, Santagata and Sandholtz (2019) evaluated whether teachers presented an appropriate pedagogical alternative and whether such an alternative would be mathematically specific. To summarize, our study used the aforementioned five analytic elements of teacher noticing (i.e., topic, actor, stance, evidence, and alternative strategy). However, the elements were further refined for this study to analyze the changes of PSTs' noticing while taking a mathematics methods course along with a practicum.

III. METHODS

1. Participants

The study's research participants were a total of 56 PSTs (15 males and 41 females) enrolled in a three-

credit Elementary Mathematics Education Methods course at a public university in Korea¹⁾. The PSTs were in the second semester of the third year out of a four-year teacher preparation program. Due to the number of PSTs, the course was divided into two sections and was taught by the same instructor (i.e., the first author of this paper). The course was compulsory for all elementary PSTs in the university. Previously, the PSTs had taken one basic course dealing with the national mathematics curriculum plus the content and pedagogy of the Number and Operation Strand. The methods course lasted 15 weeks, and the PSTs had their first four-week practicum period in the second half of the semester. As such, they did not have any formal teaching experience at the beginning of the semester. To complete 45 credit hours over the semester, the class met about four hours per week except during the practicum period.

2. Data collection

To analyze how elementary PSTs' noticing changed while taking the semester-long methods course in conjunction with a practicum, we collected the PSTs' written comments on an elementary mathematics lesson three times throughout the semester: (a) at the beginning of the course, (b) in the middle of the course after introducing the content and pedagogy of elementary mathematics teaching, and (c) at the end of the course after completing a four-week practicum and subsequent discussions on the mathematics lessons implemented during the practicum (see *Figure 1*). The specific data collection procedures are as follows.

Phase 1. In the first week, the PSTs watched a specific lesson video and wrote comments. The video was available on the official website of the Korea Education and Research Information Service. The website displayed the videos that were evaluated as good lessons by the educational offices of each province in Korea²⁾. The lesson dealt with the surface area of a rectangular prism and was for sixth graders (see *Table 1* for the lesson flow). The lesson was selected for two main reasons. First, it was relevant to the first research question of this study. The lesson was about the Measurement Strand to be covered during the methods course. The PSTs had not learned how to teach the measurement topic yet. Thus, it was appropriate to compare the PSTs' noticing before and after learning theoretical background for teaching measurement content. The second reason relates to the quality of the lesson. Although the problem in the lesson might

1) Whoever wants to be an elementary school teacher in Korea must enter one of 13 universities offering elementary teacher preparation programs. The four-year programs are similar across the universities in that PSTs must complete specific coursework of about 140 credits to be ready for teaching every subject matter of elementary school and perform about 10 weeks of fieldwork. PSTs are usually required to take two core courses specific to mathematics: One course deals with background information on teaching elementary mathematics (e.g., purpose, philosophy, or psychology of mathematics education). The other course deals with practical issues related to teaching methods along with content strands of elementary mathematics. After completing the coursework, PSTs need to pass a national teacher employment test to become a public school teacher (see Pang, 2015, for the details of elementary teacher preparation programs in Korea).

2) The educational office of each province organizes instruction-research contests for teachers and evaluates the lessons by specific criteria—for instance, lesson design, learning environment, learning objective, research topic, student activities, teaching techniques, learning summarization, goal attainment, teacher questioning, and instructional materials.

Figure 1 Timeline of data collection

Table 1 The flow of the lesson

Instructional phase	Main activities	Time (min)
Motivation for the lesson and identification of the lesson goal	<ul style="list-style-type: none"> Recall the characteristics of a rectangular prism by showing a cookie box Think about the method of how to use the least amount of paper when wrapping the cookie box Present the lesson goal: Let's explore how to figure out the surface area of a rectangular prism 	5
Exploration of multiple methods to find the surface area of a rectangular prism	<ul style="list-style-type: none"> Introduce the definition of surface area and write it in a notebook Predict how to find the surface area of the given rectangular prism Discuss methods in groups 	2 6
Presentation of various solution methods and whole-group discussion	<ul style="list-style-type: none"> Whole group discussion regarding how to figure out the surface area of the rectangular prism <ul style="list-style-type: none"> Method 1: Adding the area of each of the prism's six faces Method 2: Adding the areas of three different faces and then doubling it Method 3: (Area of the rectangular base) $\times 2$ + (Area of the side face) Calculate the surface area of the rectangular prism with the three methods above Review each method 	7 11
Generalization of the formula for finding the surface area of a rectangular prism	<ul style="list-style-type: none"> Explore the most effective method and formalize how to figure out the surface area of a rectangular prism 	5
Application	<ul style="list-style-type: none"> Solve the review exercises in the textbook 	2
Closing	<ul style="list-style-type: none"> Talk about what you have learned during the lesson Assignment: Calculate the surface area of a rectangular prism you would find in everyday life 	2

not be mathematically challenging, the teacher in the video allowed her students to explore various methods to find the surface area of a rectangular prism and to discuss the methods in small groups. The teacher then orchestrated the whole-class discussion to compare and contrast the solution methods and to generalize how to figure out the surface area of a rectangular prism.

After watching the entire lesson for 40 minutes, the PSTs were encouraged to write their comments using the following prompt: Think about the students' thinking or understanding and the teacher's instructional strategies in the video and write down whatever you found interesting. As the lesson video was publicly available, the PSTs could watch the video as often as they liked. At the end of the first week of the course, the PSTs were asked to submit, at maximum, two-pages of comments about the video.

Phase 2. After collecting the PSTs' initial comments on the video in the first week (i.e., Data 1), the instructor taught mathematical content and pedagogy related to the Geometry, Measurement, and Data and Possibility Strand of the elementary mathematics curriculum, in a lecture-oriented style. As several mathematics content strands were covered during the first half of the semester, the instructor focused on clearly explaining the content and pedagogy that the PSTs should know in teaching elementary school students. For instance, the lecture on the Measurement Strand included the need for teaching measurement, the topics and instructional remarks emphasized in the national elementary mathematics curriculum, methods and units of measurement, and common misconceptions of elementary school students related to measurable attributes. In the eighth week, to help the PSTs prepare for their first practicum, the instructor gave them a brief lecture on the Five Practices for Orchestrating Productive Mathematics Discussions, by Smith and Stein (2018). The instructor also explained instructional models addressed in teacher manuals, which are mainly used for lesson planning and implementation. The PSTs were then asked to watch the same video they had watched earlier and submit their second round of comments, complying with the same guidelines. Again, all PSTs' comments were collected (i.e., Data 2).

Phase 3. The PSTs were next assigned to nearby elementary schools according to the university's regulations for their four-week practicum period. At this point, a specific course assignment was given to connect the PSTs' teaching experience during the fieldwork with subsequent lectures. Four or five PSTs assigned in the same elementary school were grouped into a team. Each team was asked to jointly design, enact, and reflect on one mathematics lesson in the elementary school. During the fieldwork, the PSTs observed in-service teachers' teaching and gained teaching experience with multiple subjects, including mathematics. The mentor-teacher from the elementary school consulted with and evaluated their PST.

After the practicum, the PSTs returned to the university and resumed the course. During the remaining three weeks of the semester, each PST team had an opportunity to debrief on their lesson planning, implementation, and reflection. In debriefing for about 30 minutes, the PSTs were asked to show significant portions of their video-recorded mathematics lesson. The instructor then orchestrated a whole-class discussion on the lesson. The PSTs actively provided feedback to peers, often asking questions about the lesson. Building on the PSTs' various responses, the instructor raised further questions and aspects to consider in planning and implementing a mathematics lesson. In the course' final week and with the same guidelines, the PSTs submitted their final comments about the same lesson video concerning the surface area of a rectangular prism (i.e., Data 3).

In summary, the PSTs watched the same video throughout the course and wrote comments. At the

beginning of the course, the main purpose of using a lesson video was to identify what and how the PSTs would notice in the video before they had learned any specific content or pedagogy related to teaching measurement. In the middle of the course, the purpose was to explore how the PSTs' comments might change after the lecture. At the end of the course, the purpose was to analyze how the PSTs' comments might further change after their practicum in elementary schools and whole-class discussions on their enacted lessons. This data collection timescale, tailored to the PSTs' main learning experiences throughout one semester, was expected to reveal some changes of their noticing skills related to the impact of their learning experiences.

3. Data analysis

The PSTs' comments were divided into semantic units dealing with the same topic. Each unit was coded according to the following five elements resulting from the review of the teacher noticing framework: *topic*, *actor*, *stance*, *evidence*, and *alternative strategy*. We elaborate on each element as follows.

The topic element was used to analyze which pedagogical issues the PSTs focused on in the comments. We employed four main topics (i.e., Mathematical Tasks, Instructional Strategies, Learning Environment, and Mathematical Discourse), building on Pang (2011). We then subdivided each topic to capture various aspects in the PSTs' comments more precisely (see *Table 2*). For instance, the topic Instructional Strategies was subdivided into mathematical content to be taught; students' characteristics; use of teaching and learning materials; implementation of a written lesson plan, including improvisation; and assessment. The use of subtopics was intended to reveal detailed information about what the PSTs noticed and what they missed (Spizer & Phelps-Gregory, 2017).

The actor element was used to analyze whom the PSTs mainly focused on in the comments. We divided

Table 2 *Analytic elements of topic*

Topic	Subtopic	Code
Mathematical Tasks	Using tasks aligned with the lesson objective	MT1
	Using tasks tailored to students' prior knowledge and motivation	MT2
	Using tasks to promote students' mathematical thinking and communication	MT3
Instructional Strategies	Using instructional strategies appropriate for the content to be taught	IS1
	Using instructional strategies appropriate to students' characteristics	IS2
	Using manipulatives and materials to help students' learning	IS3
	Effective implementation of the written lesson plan, including improvisation	IS4
	Assessment	IS5
Learning Environment	Establishing a learning environment through appropriate social norms	LE1
	Physical arrangement and equipment to support learning	LE2
Mathematical Discourse	Mathematical communication between the teacher and students, and among students	MD1
	Meaningful questions and appropriate feedback	MD2

Table 3 *Analytic elements of actor*

Actor	Description	Code
Teacher	The comment mainly focused on the teacher in the video	T
Students	The comment mainly focused on a particular student, a group of students, or all the students in the video as a whole	S
Teacher and Students	The comment equally paid attention to both the teacher and the students	T/S

Table 4 *Analytic elements of stance*

Stance	Description	Code
Describe	Negative statement	D-
	Positive statement	D+
Evaluate	Negative statement	E-
	Positive statement	E+
Interpret	Negative statement	I-
	Positive statement	I+

it into three categories: the teacher, students, and both (see *Table 3*). Note that we did not further subdivide the category of students into whole students, a small group of students, or a particular student because our PSTs paid less attention to the students than to the teacher in the video. However, the three categories of actor can still reveal meaningful changes in PSTs' noticing skills (Santagata & Yeh, 2014).

The stance element was used to analyze how the PSTs mentioned the events they noticed in the video. Building on van Es and Sherin (2008), *stance* was further classified as describing the events in the video, evaluating what would be good or bad, or interpreting what they noticed (see *Table 4*). Moreover, each statement was judged to be either positive or negative (Santagata et al., 2007).

The evidence element was used to analyze whether or not the statements that the PSTs commented on were based on evidence. We intended to analyze not only whether the PSTs referred to specific events' evidence but also what the main sources of such evidence were. In fact, PSTs' comments were based on various sources of evidence, partly because they watched the entire lesson instead of lesson segments. These results led us to subdivide the sources of evidence, as shown in *Table 5*.

Finally, the alternative strategy element was used to analyze whether or not the PSTs proposed any alternative approaches to the event they commented on. As described in the literature review, teachers at higher levels of noticing can present alternative pedagogical solutions. In this study, the alternative strategy of the PSTs' comments was further classified as superficial or specific, and by how many alternative strategies were suggested, as shown in *Table 6*.

All the PSTs' comments (i.e., Data 1, 2, and 3) were coded according to the five analytic elements above. The coding was conducted by one researcher (i.e., the co-author of this paper) and three research assistants majoring in elementary mathematics education. Before coding, the coders were told how to

Table 5 *Analytic elements of evidence*

Evidence	Code	Evidence	Code
Students' responses in the lesson video	S	Mathematical (or theoretical) reason	M
Personal experience or opinion	E	Affective reason	A
Connections across the lesson flow	F	Learning environment	L
Decision of the teacher in the video	T	Comparison with other strategies	C
No evidence	N		

Table 6 *Analytic elements of alternative strategy*

Alternative strategy	Description	Code
Level 1	No alternative strategy proposed	1
Level 2	Vague or superficial alternative strategy proposed	2
Level 3	One specific alternative strategy proposed	3
Level 4	Two or more specific alternative strategies proposed	4

code the data, with detailed explanations for each analytical element. They practiced coding with three samples from the PSTs' comments. Note that there were 168 comments (i.e., three comments from each of 56 PSTs). To check reliability among the coders, we randomly selected 45 comments from 15 PSTs to be coded independently. The results of this coding showed more than 80% agreement for each analytic element: specifically, 90.5% in topic; 85% in actor; 80.8% in stance; 81.9% in evidence; and 95.8% in alternative strategy. After coding all the semantic units in the PSTs' comments, as illustrated in *Table 7*, the frequency was obtained for each analytic element to explore any changes in the PSTs' noticing. Representative examples of the PSTs' comments are presented in the results section to illustrate substantial changes across different time points during the semester.

IV. RESULTS

1. Changes in topic

Table 8 shows the topics that this study's PSTs noticed during the semester. The most frequent topic was Instructional Strategies followed by Mathematical Discourse, Mathematical Tasks, and Learning Environment, in that order. Comparing their noticing throughout the semester (beginning, middle, and end of the course), we found the subtle but important changes in PSTs' noticing.

The PSTs' noticing did not show major changes between the first and second comments. The total number of PSTs' comments by the semantic units increased slightly from their first to second comments

Table 7 *Example of analysis*

Excerpt from a PST's comment (a semantic unit)	Topic	Actor	Stance	Evidence	Alternative strategy
It was good that the teacher didn't mention the most effective method immediately after the students found the three solution methods. Because of that, the students had the opportunity to think of and present why the specific method was the most effective to calculate the surface area of a rectangular prism.	IS1	T/S	E+	S	1

Table 8 *Changes in topic*

Topic		Frequency (%)*		
		First comment (data 1)	Second comment (data 2)	Third comment (data 3)
Mathematical Tasks	MT1**	13 (2.92)	11 (2.26)	26 (4.30)
	MT2	21 (4.72)	24 (4.94)	30 (4.96)
	MT3	1 (0.22)	4 (0.82)	5 (0.83)
	Total	35 (7.87)	39 (8.02)	61 (10.08)
Instructional Strategies	IS1	133 (29.89)	157 (32.30)	123 (20.33)
	IS2	31 (6.97)	24 (4.94)	26 (4.30)
	IS3	66 (14.83)	51 (10.49)	73 (12.07)
	IS4	12 (2.70)	25 (5.14)	29 (4.79)
	IS5	2 (0.45)	10 (2.06)	10 (1.65)
	Total	244 (54.83)	267 (54.94)	261 (43.14)
Learning Environment	LE1	26 (5.84)	33 (6.79)	14 (2.31)
	LE2	9 (2.02)	4 (0.82)	5 (0.83)
	Total	35 (7.87)	37 (7.61)	19 (3.14)
Mathematical Discourse	MD1	70 (15.73)	65 (13.37)	119 (19.67)
	MD2	61 (13.71)	78 (16.05)	145 (23.97)
	Total	131 (29.44)	143 (29.42)	264 (43.64)
Total		445 (100.00)	486 (100.00)	605 (100.00)

*Frequency refers to the number of semantic units from comments. The percentage (%) is rounded from the third decimal place.

**MT1: Using tasks aligned with the lesson objective. MT2: Using tasks tailored to students' prior knowledge and motivation. MT3: Using tasks to promote students' mathematical thinking and communication. IS1: Using instructional strategies appropriate for the content to be taught. IS2: Using instructional strategies appropriate to students' characteristics. IS3: Using manipulatives and materials to help students' learning. IS4: Effective implementation of the written lesson plan, including improvisation. IS5: Assessment. LE1: Establishing a learning environment through appropriate social norms. LE2: Physical arrangement and equipment to support learning. MD1: Mathematical communication between the teacher and students, and among students. MD2: Meaningful questions and appropriate feedback.

(i.e., from 445 to 486). However, both sets of the PSTs' comments tended to focus primarily on Instructional Strategies (approximately 55%) and Mathematical Discourse (approximately 29%), regardless of whether they had learned the mathematical content and pedagogy related to teaching elementary mathematics. Only slight changes occurred regarding the subtopics of Instructional Strategies. Specifically, the frequency of comments about the following subtopics increased: "using instructional strategies

appropriate for the content to be taught” (IS1) and “effective implementation of the written lesson plan including improvisation” (IS4). On the other hand, “using instructional strategies appropriate to students’ characteristics” (IS2) and “using manipulatives and materials to help students’ learning” (IS3) decreased. The PSTs’ comments regarding Mathematical Discourse also slightly changed. Specifically, PSTs tended to notice the teacher’s questions and feedback more than the mathematical communication between the teacher and students, and among students.

Given these findings, at best, the university lecture seemed to lead the PSTs to focus more on the teacher’s lesson implementation and the mathematical content than on the students’ characteristics or learning. To emphasize, “using instructional strategies appropriate for the content to be taught” (IS1) was the most frequent subtopic from the first comments and this trend was reinforced in the second comments.

Whereas the changes between the first and second comments were minimal, those between the second and third were considerable. First of all, the total number of semantic units in the third set of comments was quantitatively greater (increasing from 486 in the second comments to 605 in the third). The most frequent topic was Mathematical Discourse, which increased from 29.42% in the second comments to 43.64% in the third comments. Specifically, the percentages of the two subtopics of Mathematical Discourse increased, and the subtopic “meaningful questions and appropriate feedback” (MD2) came to be the most frequently commented upon subtopic.

Along with Mathematical Discourse, the frequency of comments about the topic Mathematical Tasks slightly increased, specifically regarding “using tasks aligned with the lesson objective” (MT1). In contrast, comments on the topics Instructional Strategies and Learning Environment decreased (i.e., the former declined from 54.94% to 43.14%, and the latter from 7.61% to 3.14%). The percentages for the subtopics “using instructional strategies appropriate for the content to be taught” (IS1) and “establishing a learning environment through appropriate social norms” (LE1) notably decreased.

In summary, the main topic that the PSTs noticed was Instructional Strategies, specifically “using instructional strategies appropriate for the content to be taught.” This trend was reinforced through the lecture on the content and pedagogy of teaching elementary mathematics. However, the major change in topic occurred after the PSTs completed their practicum and discussions on their lesson planning, implementation, and reflection. The PSTs noticed more about mathematical communication, questions, and feedback as well as the content to be taught.

2. Changes in actor

Table 9 reveals the analysis of whom the PSTs mainly focused on in their comments. The comments of the PSTs tended to focus on the teacher in the lesson video. More than 80% of the comments in both the first and second sets focused on the teacher. However, in the third set of comments, the PSTs’ noticing

Table 9 *Changes in actor*

Actor	Frequency (%)		
	First comment (data 1)	Second comment (data 2)	Third comment (data 3)
Teacher	358 (80.45)	404 (83.13)	269 (44.46)
Students	80 (17.98)	62 (12.76)	86 (14.21)
Teacher and Students	7 (1.57)	20 (4.12)	250 (41.32)
Total	445 (100)	486 (100)	605 (100)

concerning the teacher dramatically decreased from 83.13% to 44.46%. Instead, there was an increase in the instances where they noticed the teacher and students together, from 4.12% to 41.32%. The PSTs' practicum and subsequent discussions led them to attend to the teacher and students simultaneously.

The following excerpts illustrate how the actor element changed from one PST's first comment to the third.

Although the student looked for another method that the teacher had not anticipated and that was difficult to display with the materials the teacher had prepared, the teacher's skillfulness was outstanding in that she was not embarrassed at all and responded appropriately to explain the method. (PST 25, first comment)

One student presented an answer unexpected from the teacher during the lesson. The student found the surface area of the given rectangular prism by unfolding it into the planar figure and calculating the area of several rectangles shown in the planar figure. This is an application of the method for finding the area of various figures that the student had learned in fourth grade. As I understand it, one of the reasons for this student to use the specific method was because the teacher provided the students with the rectangular prism in a form of its nets. The teacher praised the method she had not anticipated, but at the same time mentioned the limitation of the method by saying "you always must unfold the rectangular prism and check it in order to use this method." (PST 25, third comment)

In the first comment, PST 25 described the teacher's skillful response to a student's unexpected answer. PST 25 did not mention this event in the second comment. In the third comment, PST 25 attended to the student's answer, inferred how it was derived, and connected the student's answer to the teacher's instructional strategies. This change is meaningful in that the PST attended to the relationship between teacher pedagogy and student thinking.

3. Changes in stance

Table 10 shows our analysis of the PSTs' comments by stance. The overall trend suggests that the PSTs' comments were generally evaluative and positive in nature. A closer look at Table 10 indicates some changes throughout the course. First, the descriptive stance tended to decrease gradually from the first to the third comments (38.20%→26.54%→14.88%), and it was generally associated with positive aspects. Next, the evaluative stance is noteworthy because it showed the highest frequency among all three sets of comments. Specifically, in comparison with the first and the second comments, the percentage demonstrating an evaluative stance increased noticeably in the third comments (42.92%→45.27%→62.64%), often along with negative evaluation (10.11%→15.64%→27.11%). We can infer that the PSTs came to evaluate the events in the lesson video more critically based on their discussions of lesson planning, implementation, and reflection during the latter part of the methods course. Finally, the interpretative stance showed the lowest percentage in the first comments (18.88%) but showed slight increases in the second (28.19%) and the third (22.48%) comments.

The following excerpts illustrate how the stance element changed from one PST's first comment to the second comment.

At the end of the lesson, the teacher gave an assignment to the students to find the surface area of a rectangular prism in everyday life and to write it down in a math journal. *This assignment seems to be an original task that students could review what they had learned using their math journals* (PST 34, *italics in original*, first comment)

This lesson is closely linked to real life. The teacher connected real life not only to the lesson activities for motivation and wrap-up but also to the assignment to find the surface

Table 10 Changes in stance

Stance		Frequency (%)		
		First comment (data 1)	Second comment (data 2)	Third comment (data 3)
Describe	Negative	39 (8.76)	41 (8.44)	20 (3.31)
	Positive	131 (29.44)	88 (18.11)	70 (11.57)
	Total	170 (38.20)	129 (26.54)	90 (14.88)
Evaluate	Negative	45 (10.11)	76 (15.64)	164 (27.11)
	Positive	146 (32.81)	144 (29.63)	215 (35.54)
	Total	191 (42.92)	220 (45.27)	379 (62.64)
Interpret	Negative	20 (4.49)	42 (8.64)	52 (8.60)
	Positive	64 (14.38)	95 (19.55)	84 (13.88)
	Total	84 (18.88)	137 (28.19)	136 (22.48)
Total		445 (100)	486 (100)	605 (100)

area of a rectangular prism that can be found in real life and write it down in a math journal. *This assignment allows students to recognize that the concept of measurement is used in their lives beyond the classroom. This is related to the mathematization by Freudenthal. Through these activities and the assignment, students can better understand what they have learned on that day and develop a sense of volume by actually measuring the surrounding diverse rectangular prisms.* (PST 34, italics in original, second comment)

In the first comment, PST 34 attended to the homework given by the teacher at the end of the lesson and evaluated it as an original task to review what was learned. In the second comment PST 34 attended to the same homework, finding a rectangular prism in daily lives and calculating its surface area in the mathematics journal. More importantly, the PST interpreted the homework from various perspectives with mathematical terms, such as “measurement,” “sense of volume,” and “mathematization,” which were learned from the lecture. Similarly, after learning the content and pedagogy of teaching elementary mathematics, some PSTs attempted to interpret the tasks or events of the lesson video based on the theories they had learned in the methods course.

4. Changes in evidence

Table 11 shows the analysis of various sources of evidence in which the PSTs' comments were grounded. The top three sources used in the PSTs' comments were their personal experience or opinion, the decision of the teacher in the video, and the students' responses in the lesson video. A closer look at Table 11 indicates some changes in evidence sources according to when the comments were completed. In the first comments, when the PSTs offered any evidence at all, they tended to base their comments more on the top three sources mentioned above: personal experience or opinion (26.52%), the decision of the teacher

Table 11 *Changes in evidence*

Evidence	Frequency (%)		
	First comment (data 1)	Second comment (data 2)	Third comment (data 3)
Students' responses in the lesson video	69 (15.51)	59 (12.14)	80 (13.22)
Personal experience or opinion	118 (26.52)	127 (26.13)	145 (23.97)
Connections across the lesson flow	14 (3.15)	19 (3.91)	22 (3.64)
Decision of the teacher in the video	83 (18.65)	85 (17.49)	156 (25.79)
Mathematical (or theoretical) reason	8 (1.80)	29 (5.97)	67 (11.07)
Affective reason	15 (3.37)	13 (2.68)	22 (3.64)
Learning environment	40 (8.99)	23 (4.73)	27 (4.46)
Comparison with other strategies	0 (0.00)	16 (3.29)	24 (3.97)
No evidence	98 (22.02)	115 (23.66)	62 (10.25)
Total	445 (100)	486 (100)	605 (100)

in the video (18.65%), and students' responses in the lesson video (15.51%). Furthermore, a considerable number of the PSTs did not present any evidence for their analysis (22.02%).

Regarding the evidence of analysis, the second comments were remarkably similar to the first comments: The PSTs' second comments were grounded in personal experience or opinion (26.13%), the decision of the teacher in the video (17.49%), and students' responses in the lesson video (12.14%). The percentage of PST offering no evidence was still high (23.66%). Nevertheless, a few minor changes occurred. There were increases in the percentages of PSTs using mathematical (or theoretical) reason (1.80%→5.97%) and comparison with other strategies (0.00%→3.29%). Also, there was a decrease in the percentage of PSTs referring to the learning environment (8.99%→4.73%). We can infer that the PSTs increased their efforts to interpret classroom events based on the theoretical reasoning or instructional strategies they had learned in the methods course. For instance, as shown below, PST 21 at first evaluated the teacher's monitoring activity during students' small-group discussions by stating that it was good. But PST 21 did not specify why such an activity would be effective. In contrast, PST 21 later provided details to explain her thinking by citing the monitoring practice among the five practices she had learned in the methods course.

It was good to see that the teacher actively asked about students' activities and summarized them while walking around during small group discussions. (PST 21, first comment)

When the small group discussions did not go on well, the teacher asked students a few questions to facilitate the discussion. This situation reminds me of the monitoring practice of 5 practices I learned in the lecture. However, among the groups caught on the camera, there was one group of students who did not discuss actively, and it was regrettable that the teacher only said "little bit more discussion ..." and passed by without any further feedback. It would have been better if the teacher could have given the group more specific feedback on how to proceed with the discussion. Considering the 5 practices, selecting in advance whom to present while monitoring students' activities would be helpful for the teacher to lead the lesson smoothly and to facilitate an effective discussion. By doing so, the students' understanding could have been enhanced. (PST 21, second comment)

The PSTs' third comments were grounded in the decision of the teacher in the video (25.79%), personal experience or opinion (23.97%), and students' responses in the lesson video (13.22%). Note that the percentage referring to the teacher's decision notably increased. In a similar vein, the percentage centered on mathematical reason increased (5.97%→11.07%). In addition, the percentage of comments that offered no evidence for analysis considerably decreased compared to the percentages in the first and the second sets of comments (22.02%→23.66%→10.25%). It is meaningful that more PSTs could provide evidence to support their analyses at the end of the semester.

5. Changes in alternative strategy

Table 12 shows the analysis of the alternative strategies that the PSTs suggested in their comments. The PSTs tended to refrain from presenting alternatives, as evidenced by the fact that more than 60% of the comments did not include any alternative strategy. Nevertheless, the percentages of comments lacking alternatives decreased slightly from the first to the third comments (71.24%→66.87%→62.15%). The percentage of comments that included a superficial alternative strategy and the percentage that included two or more specific alternative strategies slightly increased from the first comments to the second (12.58%→14.81% for the former; 1.80%→3.50% for the latter). Likewise, the percentage presenting one specific alternative strategy increased from the second comments to the third (14.81%→22.98%).

Despite the minor changes, it is encouraging to see the PSTs' tendency to propose specific alternatives for the events they noticed in the lesson video while taking the methods course (see below for illustrative excerpts).

It seemed that the number of students presenting in the lesson was limited. I could see some students continuing to present, so I thought that only the students who were presenting were doing it. Of course, it is impossible for all students to present in one lesson, but I think it is good for the teacher to give many students a chance to present. (PST 19, first comment)

When the students were writing on the board how to find the surface area of the cookie box, it was regrettable that the teacher asked the student who had finished writing to present his method first, because other students were still writing. It would be helpful for the student who solved the problem with the first method to listen to how to solve it with the second, third, and fourth methods. As the teacher asked the student who first solved to present, however, the students who solved the problem late could not listen to other students' presentations. (PST 19, second comment)

It would be better to revise the order of students' presentations on how to find the surface

Table 12 *Changes in alternative strategy*

Alternative strategy	Frequency (%)		
	First comment (data 1)	Second comment (data 2)	Third comment (data 3)
No alternative strategy	317 (71.24)	325 (66.87)	376 (62.15)
Superficial alternative strategy	56 (12.58)	72 (14.81)	69 (11.40)
One specific alternative strategy	64 (14.38)	72 (14.81)	139 (22.98)
Two or more specific alternative strategies	8 (1.80)	17 (3.50)	21 (3.47)
Total	445 (100)	486 (100)	605 (100)

area of the cookie box. The teacher led the students to present in order of (a) adding the areas of the three pairs of congruent faces, (b) adding the side area and the base area, (c) adding the areas of each face, and (d) dividing the net of the box into parts and adding their areas. *I think it is right to make presentations in the order of (c), (a), (b), and (d), because adding the areas of each face requires only a simple computation, but the two methods (i.e., adding the side area and the base area, and adding the areas of the three pairs of congruent faces) require more advanced thinking processes in that they are connected to the properties of a rectangular prism, such as “two faces facing each other are congruent” or “the total width of the side and the sum of edges of the base are the same”.* To this end, it would have been better if the teacher had made a note of students' solution methods while walking around (*monitoring*) and led students to present on how to find the surface area of the cookie box in pre-selected order (*selecting*). (PST 19, *italics in original*, third comment)

In the first and second comments, PST 19 only mentioned the limited number of students making presentations in the video and the order of presentations on how to find the surface area of a rectangular prism. In contrast, in the third comment, PST 19 mentioned specific alternatives for how to present students' ideas. In particular, PST 19 considered students' thinking process in calculating the surface area of a rectangular prism and suggested an alternative strategy based on monitoring and selecting among the five practices she had learned during the methods course.

V. DISCUSSION AND CONCLUSION

1. Changes in teacher noticing through a mathematics methods course

The PSTs improved their noticing skills while taking an elementary mathematics methods course that combined both theoretical and practical aspects of teaching mathematics. On the one hand, this result aligns with prior studies that reported PSTs' enhanced ability to notice (e.g., Mitchell & Marin, 2015; Schack et al., 2013; Star & Strickland, 2008; van Es et al., 2017). On the other hand, our study's result can be differentiated from prior studies using a popular pre-post measure in analyzing the development of noticing (Amador et al., 2021). Note that this study traced the changes in the PSTs' noticing in a way that was tailored to their learning experience for one semester. This methodological approach, with the support of a detailed analytical framework, is helpful for unpacking changes in what PSTs notice and how they notice.

Compared to the first comments that the PSTs wrote at the beginning of the methods course, their second comments about the same lesson video indicate that their noticing ability was enhanced. Specifically, the PSTs tended to focus more on mathematical content and the teacher's lesson

implementation. Note that the most frequent subtopic was the use of instructional strategies appropriate to the content to be taught. The PSTs also tended to analyze the lesson in an interpretative stance based on mathematical or theoretical reasons or teaching strategies they had learned in the methods course.

Note that this study's methods course was not designed specifically to improve teacher noticing. Even though the course covered the Measurement Strand of the elementary mathematics curriculum, it did not elaborate on how to teach the surface area of a rectangular prism. However, the results indicate positive changes in teacher noticing through a lecture on the content and pedagogy related to teaching elementary mathematics in general, even though such changes were minimal in this study.

Compared to the first and the second comments, the third comments the PSTs made at the end of the semester illustrated substantial changes in their noticing abilities across all analytic elements: (a) they focused more on Mathematical Discourse beyond Instructional Strategies (*topic*); (b) they came to see the teacher and students together (*actor*); (c) they tended to evaluate the classroom events more as they noticed (*stance*); (d) they used more evidence to support their analyses (*evidence*); and (e) they proposed specific alternatives to the classroom events they noticed (*alternative strategy*). These changes arose after the PSTs' practicum and subsequent discussions as a part of the methods course.

It is common that a mathematics teacher preparation program includes at least one methods course and the practicum. Note that the methods course of this study connected the lecture-oriented university program to the practicum at elementary schools through a group assignment of lesson planning, implementation, and reflection. The practicum itself may have provided a meaningful experience for PSTs, but just participating in the fieldwork experience does not increase teacher noticing ability (Santagata & Yeh, 2014). In contrast, the group assignment during the practicum period and subsequent discussions on it in the methods course seem to have positively impacted the PSTs' noticing ability.

Discussing a video-taped lesson among teachers, usually including a teacher educator or facilitator, is a common pedagogical approach that teacher education programs employ to improve teacher noticing (McDuffie et al., 2014; Mitchell & Marin, 2015; van Es & Sherin, 2008). However, unlike many other studies in which discussions centered on video clips selected by researchers, the PSTs in this study discussed in small groups how they planned and enacted their mathematics lessons. They then had an opportunity to discuss further with the instructor and other PSTs from multiple perspectives. The PSTs' actual experience of teaching mathematics and sharing diverse interpretations seem to have improved their noticing ability (Fernández, Llinares, & Rojas, 2020; Lee & Choy, 2017).

This study shows that PSTs' noticing ability can be significantly enhanced through a mathematics methods course combining theory and practice. Our study found that the degree and aspects of positive changes in teacher noticing were different depending on when the PSTs' comments were made. Our findings suggest that there is much room for further development of teacher noticing because even the third round of participant comments provided more evaluative comments than interpretive comments, frequently without alternative strategies. Nevertheless, it is encouraging that one semester-long methods

course led PSTs to improve their ability to notice salient features of a mathematics lesson.

2. Teacher noticing analytic framework

In this study, we used the five elements as an analytic framework—*topic*, *actor*, *stance*, *evidence*, and *alternative strategy*—based on a review of prior studies on teacher noticing. The elements led us to explore how our PSTs' noticing ability changed while taking a specific methods course. What follows are implications of the noticing framework, specifically regarding *topic* and *evidence*, which were the elements we analyzed in more detail.

Regarding the analytic element *topic*, many studies have investigated the tendency of PSTs to notice a given topic and the frequency with which the topic is noticed. Most studies have focused on broad topic categories, such as mathematical content, classroom environment, or mathematical discourse (Star & Strickland, 2008). This study also included such broad categories but further refined them into a total of 12 subtopics to look closely at changes in teacher noticing. Such subtopics led us to explore in detail what the PSTs exactly noticed. Note that there were remarkable similarities among the four main topics between the first and second comments the PSTs made. The subtle but salient differences appeared when we analyzed the subtopics apparent in the comments. For instance, the percentages of comments dealing with Instructional Strategies were almost the same in the first and second sets of comments. However, the analysis by Instructional Strategies subtopics indicated that the PSTs focused more on using strategies that are appropriate for the content to be taught rather than using strategies that are appropriate to students' characteristics. In a similar vein, the subtopics helped us pin down where the main differences between the second and third comments arose from. For instance, the big difference concerning Mathematical Discourse resulted from each of the two subtopics (i.e., MD1 and MD2). However, the subtle difference concerning Mathematical Tasks resulted from one of the three subtopics, MT1.

The detailed analytic framework using subtopics also helped us explore which features need to be emphasized in a teacher education program. For instance, it is encouraging that the PSTs focused more on mathematical tasks while taking the methods course. However, their focus was centered either on using tasks aligned with the lesson objective or on using tasks tailored to students' prior knowledge and motivation. On the contrary, little focus was on using tasks to promote students' mathematical thinking throughout the semester. Knowing exactly what the PSTs noticed and what they missed is helpful for re-designing the methods course (Spizer & Phelps-Gregory, 2017).

Regarding the analytic element *evidence*, many studies have analyzed whether or not PSTs could provide detailed or convincing evidence when interpreting the classroom events they noticed (Jacobs et al. 2010; Santagata & Sandholtz, 2019; van Es, 2011). Our study specified the sources of evidence in detail based on the PSTs' comments. Because the PSTs in this study were asked to write their comments after watching an entire mathematics lesson, not a short video clip, they employed various reasons to defend

their comments, such as personal experience, the decision of the teacher in the video, students' responses in the video, or mathematical reasons the PSTs had learned. The specified sources of evidence helped us analyze the basis upon which the PSTs evaluated or interpreted classroom events, and how such a basis might change throughout the PSTs' university studies.

In fact, it was encouraging that the PSTs' comments in this study were grounded in the decision of the teacher in the video as well as their personal experiences or opinions. Their comments were also based on more mathematical reasons as the semester went on. Given that teachers are expected to discuss their lessons throughout their teaching career, any teacher preparation program needs to foster the ability to evaluate or interpret classroom events based on specific and reasonable source.

To summarize, this study devised a more detailed framework to analyze PSTs' noticing ability, building on prior studies, specifically regarding the actor and evidence elements of analysis. Such a framework broadens our understanding of the changes in teacher noticing throughout a mathematics methods course localized in the Korean context. It is hoped that this study's analysis of teacher noticing will provoke discussions both on how to design a mathematics methods course in a teacher preparation program and on how to measure noticing as an indicator of teacher expertise.

CONFLICTS OF INTEREST

No potential conflict of interest relevant to this article was reported.

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