Original Article

An Investigation of Student Participation in Collaborative Problem Solving in Mathematics: Positioning and Negotiation among Four Chinese Students

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Abstract

In this paper we report on a case study involving four Chinese students; the aim of the study was to develop a conceptual framework for investigating student participation in a collaborative task in mathematics. Building on previous research on student participation, we defined student participation in a collaborative task in mathematics as the process of taking part in student interactions and task completions. A video recording and transcript of students involved with collaborative task in mathematics were analyzed. Four categories (initiation, response, evaluation, and non-interactive) were created for analyzing the students' interactions. The four students showed different approaches to communicating with other group members. The content of the group's discussion was examined in terms of the negotiation process. By looking at the content of the group's discussion, the process how the group work on the task could be revealed. We identified that students showed involvement in the group discussion by proposing and revisiting topics. It was found that through forming an interactive pair, students might be able to take up the leading role in working on the task and controlling the conversation of the group.

Keywords: collaborative task in mathematics, negotiation, participation, positioning, student interaction

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I. INTRODUCTION

Recent curriculum and instruction reforms have put a greater emphasis on the teaching and assessment of 21st century skills. For example, the recent Future of Education and Skills 2030 project initiated by OECD has involved around 25 countries to conduct a cross-national study of curriculum to incorporate the 21st century skills, including communication, problem solving and so on, in curriculum and instruction (OECD, 2020). Collaborative problem solving is regarded as critical and necessary for effective functioning in society in the 21st century, by the governments, industry and educational theorists, thus, collaborative problem solving skills have also been seen as essential 21st century skills (Griffin et al., 2012; OECD, 2017). To improve students' collaborative problem solving skills, collaborative tasks have been increasingly advocated as a daily classroom practice in many countries (e.g., ADET, 2012; MOE, 2011; NCTM, 2000). In collaborative tasks, individuals are given opportunities to develop their skills in collaboration and communicating among group members, understanding and assigning roles, and to maintain and adapt to achieve the goals of the group. Unlike problem solving activities that are carried out individually, collaborative tasks are more complex because different students within a group may be involved in and contribute to the activity in different ways. To improve students' collaborative problem solving skills, it is essential to first understand how students participate and behave in the context of collaborative tasks.

In China, according to the 2011 national mathematics curriculum standards, teachers are required to teach collaborative tasks in mathematics as part of their everyday lessons. However, this recent introduction does not mean that all teachers are equipped with the skills to follow the standard. Teaching collaborative tasks in mathematics is not only a challenge for teachers in China, but for those in other countries (Chan & Clarke, 2017; Langer-Osuna, 2016). We propose that student participation may be important to understanding how students work on collaborative tasks in mathematics, so as to help teachers provide effective and timely instructions. In this study, we examined the process by conceptualizing student participation in terms of the roles that students assumed during a problem-solving activity and the content of their contribution to the discussions. Based on this we addressed the question: What could participation look like in a collaborative task in mathematics? We begin by explaining the ways in which student participation has been studied and defined in the past and the theoretical frameworks on which this study is drawn.

II. LITERATURE REVIEW

The study of student participation has been of interest to educational research; participation and learning have been written about in similar ways in the literature. Sfard (1998) proposed that learning can be viewed in terms of two metaphors: acquisition and participation. The acquisition perspective focuses on individuals' internal states, including what knowledge and competency they have gained as

an outcome. In contrast, the participation perspective regards learning as an activity. This view focuses on the group dynamics and the community of practice in which individuals slowly become familiar with the culture or norms within the community. Sfard (1998) wrote that both metaphors are needed because there are no “one-for-all practical recipes” (p. 10) for teaching and learning. There is a need to consider acquisition and participation in educational research and practice because they address different aspects of learning and teaching.

Similarly, when looking at the concept of participation, there is a parallel between how participation has been viewed and how learning has been viewed. The Oxford English Dictionary defines participation as, “the action of taking part in something.” According to a socio-constructivist perspective, Lave and Wenger (1999) proposed that participation refers to the process of becoming part of a community of practice in which learners are increasingly taking on more responsibilities. This view emphasizes participation as a process. Although Hesse, Care, Buder, Sassenberg, and Griffin (2015) also mentioned the socio-constructivist perspective of participation in their paper, they mainly defined participation as a set of skills that students have, which is more akin to a behaviorist perspective (Shepard, 2000). In their paper, they distinguished three aspects: action, interaction, and task completion, when assessing participation in a collaborative problem solving context. Action was described as referring to the “general level of participation of an individual, irrespective of whether this action is in any way coordinated with the efforts of other group members” (p. 42); interaction was defined as referring to “behavior that demonstrates interaction with and responses to others” (p. 44); and task completion was explained as referring to “motivational aspects of participation and consequent perseverance on a task” (p. 44). In contrast with Lave and Wenger’s perspective in which participation is viewed as a process, Hesse et al.’s (2015) focus was on assessing student performance in collaborative problem solving activities as a set of skills that students can acquire over time.

We believe that drawing from both perspectives (behaviorist and socio-constructivist) can provide a more comprehensive understanding of participation. Drawing from two particular aspects of student participation raised by Hesse et al. (2015) in terms of interaction and task completion, in this study, rather than viewing student participation as a set of skills, we viewed it as a process. Student participation was defined as the process of taking part in student interactions and task completions, through the taking on of different roles and being involved in the discussion of different topics during problem solving activities. By this definition, collaborative tasks in mathematics is treated as a classroom context where students are assigned an open-ended task and to complete the task as a group naturally, hence, different with some previous studies, first, we consider students’ participation in the process of a collaborative task in mathematics as how they solve and complete the open-ended mathematics task as a group, ‘complete the task’ is then treated as equally as ‘solve the problem’ in such a context; second, we do not tend to view collaboration as an outcome or production of joint efforts (e.g. Staples, 2007; Andrà et al., 2020), but aim to capture the nature characteristics of collaborative task completion in mathematics.

1. Student interaction and the content of group discussion

In Stasser and Vaughan's (1996) review of research in group participation during face-to-face, unstructured discussions, many of the models of participation reviewed were about how group members took turns during discussions. Turn taking was operationalized in terms of how frequently a person would speak and when and to whom a person would speak (e.g., Parker, 1988; Stephen & Mishler, 1952; Stasser & Taylor, 1991). Despite this common operationalization, turn taking does not give us information about the ways in which a person contributes to a discussion. In the context of a collaborative task in mathematics, even though one group member could take a turn to speak with others, this did not necessarily mean the person made a useful contribution. We therefore attempted to develop a conceptual framework of student participation to go beyond turn taking.

Two concepts seemed to be particularly useful: One was student interaction and the other was the content of discussion. Student interaction is about who is communicating with whom, while the content of discussion is about what is being communicated. Student interaction refers to reciprocal communications, either verbal or nonverbal, among students. For example, one may initiate a conversation by asking a question while another may or may not respond by answering the question. In terms of task completion in mathematics, the discussion content could be about strategies, concepts, or information related to the problem; task allocation among group members; and off-task topics. For students working collaboratively to solve mathematical problems, certain contents of the discussion might encourage or inhibit student interactions. For example, Wood (2013) found that students may have different preferences toward mathematical or non-mathematical content during collaborative activities and may participate in the discussion differently depending on the focus of the discussion. In this study, in addition to examining student interactions, the content of the discussion in relation to the task completion as well as the contribution of individual students to particular discussions were also considered.

1) Studying student interaction and the use of positioning theory

In educational research literature, there appears to be a tendency for student interactions to be viewed as a means to an end. Some researchers are interested in what kind of student interactions may lead to good collaboration outcomes (e.g., Barron, 2003; Cohen, 1994). Some propose that through talking about mathematics and interacting with each other during the activity, students have the opportunity to learn mathematical concepts and improve their mathematical reasoning abilities (e.g., White, Wallace & Lai, 2012). The authors of such studies usually treat the process of interaction as a collection of broad conditions leading to a particular outcome, while what is going on within the individual groups during the task completion activity has not been studied in detail. Scholars recently examined students' positioning and positions within the group during collaborative discussions (Bishop, 2012; DeJarnette, 2018; Esmonde, 2009; Turner et al., 2013; Wood, 2013). This line of research, which mainly aims to identify students'

interactional patterns, may be useful for understanding the mechanism of how students interact with each other during group discussion to solve mathematical tasks.

Positioning theory offers a way to understand social interactions by examining how people take up places (positions) in a social setting and make their voice heard and understood by other people. There might be implicit or explicit rules that people follow in order to speak and be listened to; these can be culturally or locally determined. For example, as part of the Learner's Perspective Study which involved comparisons of eighth-grade mathematics classrooms in 16 countries, Xu and Clarke (2019) found that in the Korean (Seoul) classrooms, the average number of student public utterances were much lower than that in the Australian (Melbourne) classrooms. However, the Korean students were observed to be involved in choral response, which was not observed in the Australian classrooms. This may suggest that culturally, it is more acceptable for Korean students to speak publicly in the classroom only as part of choral response, while this is not the case in Australia.

According to positioning theory, there are rights and duties associated with positions, such as the right to speak or the right to hear (Barnes, 2003). In collaborative tasks in mathematics, examining changes in the rights and duties of group members allows us to understand the explicit or implicit rules of student interactions during group activities. Some researchers (Anderson, 2008; DeJarnette, 2018; Esmonde, 2009; Wood, 2013) draw upon positioning theory to understand how the students' positions shift in the process of collaborative task completion. In this study, we applied positioning theory to investigate how students interacted with each other in terms of what role (position) students they assumed during the group discussion (that is, whether a student initiated a discussion topic or responded to others' comments). The term position in this paper refers to the role that a student played within a group during the discussion of a specific topic. The use of the word as a verb, to position or positioning, denotes the process of occupying a position. While student interactions within a group could either be verbal or non-verbal (i.e., involving gestures), in this study, we mainly focused on the verbal interaction among students. Non-verbal interactions were referred to for supplementing interpretation of verbal interactions.

According to positioning theory, in a social context, people may use different actions (e.g., asking a question, making a claim) to take up positions (Harré & van Langenhove, 1999). Researchers may infer the position a person has based on their observed actions and those of other group members (e.g., Esmonde, 2009). Labels such as expert or novice and leader or follower are sometimes used to denote a person's position. This can be problematic as the labels may imply a particular inherent status, for example, that experts are better than novices, or leaders are better than followers. By focusing on the actions of the participants, we were aware of the dangerous of referring the inferred statuses and chose to avoid being preferential toward a particular position during the analysis. Thus, our analysis of student interactions focused on the particular verbal actions students took and how those actions might have created roles that students could play during the task completion activities.

2) Studying the content of group discussion

Even though positioning theory provides an approach to examining student interactions in a collaborative task in mathematics, the approach does not take into account the process of how a group complete the task as a group. In order to investigate what was being communicated and how students were involved in the task completion process, in addition to studying student interactions, we also examined the content of the group discussion.

Other than viewing student participation as the process of taking positions during student interactions, the process of participation in group discussion could also be viewed as the process of negotiation of meaning. When students are involved in group discussion, they may understand a topic in different ways. Through the process of discussion, they come to understand each other's perspectives and ideas. The term negotiation of meaning is used to conceptualize this process of socialization in which people negotiate between their own understanding and that of others. Clarke (2001) characterized negotiation as a "cyclic process of refraction (construal), reflection, and representation, the goal of which is consensus" (p. 35). Through this process, group members may make an effort to understand each other's utterances and build on each other's ideas. When considering group discussion as a negotiation process, what is being negotiated within the group indicates the focus of the discussion and the common goal (e.g., solving a problem) that is shared by the group members.

In the literature, the negotiation process can be viewed from two different perspectives. Some view it as a social process in terms of the social influence of negotiation within a group, referring to content that was exchanged and refined through the process. For example, DeJarnette (2018) viewed negotiation as a process in which students take turns speaking to exchange knowledge and actions within the group. Instead of looking at what is shared within the group, some researchers may concentrate on an individual's personal domain during the negotiation process (see Clarke, 2001). For example, Engle, Langer-Osuna, and McKinney de Royston (2014) considered to what degree personal domains could be negotiated when individuals are situated in a social context. In this study, the two perspectives of viewing negotiation (social and personal) were not seen as dichotomous; both perspectives were taken into account. Focusing on the negotiation process during collaboratively completing tasks in mathematics allows us to look not only at the shared purpose of group discussion but also at each student's involvement in the discussion.

Positioning theory and the concept of negotiation appear to be complementary when conceptualizing student participation in a collaborative task in mathematics. Positioning theory focuses on analyzing the roles that different students play during group discussions while viewing group discussion as a negotiation process allows researchers to examine the content of the discussion. Therefore, we designed this study to draw from these theoretical lenses. Our aim was to integrate the underpinnings of these lenses to form a more comprehensive conceptual framework for the issue at hand. We asked the question, "What could student participation in a collaborative task in mathematics look like in terms of students taking different roles (positions) and being involved in the discussion during task completion activities?" We tried to answer

the question through answering the following sub-questions:

RQ1: How did each student in the group interact with each other and play their roles during the task completion process based on their verbal utterances?

RQ2: How did individual students involve themselves in the discussion of each topic during the task completion process?

Through answering the two research questions, we were able to operationalize the conceptual framework and see how it could be used to understand student participation in practice. The framework may help teachers improve their instruction in collaborative tasks in mathematics by having a more nuanced understanding of student participation during such activities.

III. METHODS

1. Data source

The data reported in this paper came from the Australian government-funded research project titled The Social Essentials of Learning (Chan, Clarke, & Cao, 2018). As part of the project, classes of Year 7 students in Australia and China were given separate open-ended tasks to complete in small groups (four to six students). The task completion process of each group was video recorded and transcribed and students' written works were collected for analysis. Since this study aimed to test a conceptual framework integrating multiple theoretical lenses, a group that demonstrated various forms of interaction was determined to be the most suitable for this purpose. One group of four students from an urban city in China was chosen. In this group were two boys (S1 and S2) and two girls (S3 and S4), all of whom were aged 13. The group

Figure 1 Task instruction provided and group solution of one student group

- 小明的公寓**
- 小明的公寓有五个房间，而公寓的总面积是 60 平方米。
- 1) 请画一幅图展示小明的公寓。
 - 2) 标注其中每一个房间的功能及尺寸（长和宽）。

Xiao Ming's apartment has five rooms, the total area is 60 square meters.

- 1) Draw a possible plan of Xiao Ming's apartment;
- 2) Label each room in the apartment and show the function and dimensions (length and width) of all rooms.

completed the task after a rich discussion. They worked on one problem solving task for 15 minutes and 42 seconds. The instruction of the group task and the group's solution are translated and shown below (Figure 1).

2. Analytical approach

In order to test the integration of theories, several steps were devised to examine both positioning and discussion topics evident in the data:

- 1) We partitioned the transcript into Negotiative Events (NEs) and identified the topic proposer for each NE;
- 2) We identified the students' interactive patterns in each NE; and
- 3) We constructed narratives to describe what positions each student took up during the discussion of a variety of topics.

These steps were developed through repeat examinations of the data, testing different units of analysis appropriate both to the constructs (positioning and negotiation) and the data (student interaction and content of group discussion). The second and third steps were chosen to answer the first research question (students' interactions and positions); the first and the third steps were selected to answer the second research question (students' involvement in different group discussion topics).

1) NE analysis

NE was defined by Chan and Clarke (2017) as "an utterance sequence constituting a social interaction with a single identifiable purpose" (p. 4). In this study, two characteristics of the NEs were considered: the specific topic(s) that the group talked about and the person who proposed the topic(s). Different individuals proposed different topics, which the group worked through to achieve specific intermediate goals. Tracing the topics that the group worked through helped us identify changes in the focus/foci of the discussion.

To identify NEs, the transcript was partitioned and the point in the transcript where there was a change in topic in the discussion was identified. If there was more than one point of focus during the discussion or if some of the students did not share the same focus as the others, the topic that was discussed by two or more group members was treated as the focus. If the group had split into two halves, where one pair of students was focusing on one topic and another pair was focusing on another topic, the foci of both pairs formed the focus of the NE. The researcher then referred to students' gestures, pauses, and other non-verbal communications from the video recording to ensure that the change of topic aligned with what was identified in the transcript. An example of an NE is shown in Table 1.

2) Students' interactive pattern analysis

Next, we examined the students' verbal interactions in terms of how they interacted with each other

Table 1 An example of NE changing (NE4 to NE5)

NE4		Topic: Whether 2 m ² is big enough for a toilet?
S4 (NE4.1):	A 2 m ² bathroom is OK.	
S2 (NE4.2):	That's impossible. I tell you.	
S3 (NE4.3):	Possibly? No. 2 m ² is impossible. If 2 m ² , even a toilet bowl is not enough.	
S1 (NE4.4):	How is it impossible?	
S2 (NE4.5):	Firstly, you should have a...	
S4 (NE4.6):	Then 5 m ² .	
S2 (NE4.7):	Firstly, it must have a place to wash. In a toilet, you must have a place to wash. A toilet should be as big as the teacher's platform.	
S1 (NE4.8):	No. No. No. You don't know, some hotels have a place to wash. It is just like, it is directly located there...	
NE5		Topic: Whether 5 m ² is big enough for a toilet?
S4 (NE5.1)	5 m ² is actually around that area.	
...	...	

during each NE. We drew from the widely used initiation-response-evaluation (IRE) model (Cazden, 2001; Mehan, 1979) to analyze students' interactions through their actions (utterances). In group discussion about specific topics, a student could initiate a topic discussion, another student then could respond to or challenge the person who initiated the topic discussion. Other than initiating and responding, students in a group may have also responded by evaluating the known information or the quality of the group conversation. The evaluation utterance could also be considered as a response to the initiating or responding utterances. In addition to initiation, response and evaluation, a person might also have chosen to not interact with the group; for example, he or she may have engaged in self-talk or may not speak at all, and therefore, we code his or her interaction as non-interactive in group discussions. These four categories provided a way to describe student interactions in terms of who was communicating with whom. It is also worth noting that the reason evaluation was considered separately from response was that in collaborative tasks in mathematics, students who contributed to evaluation can play an indispensable role in encouraging or sometimes even inhibiting the task completion process. Applying a more dynamic perspective, the four categories are not fixed but may change during the course of the interaction. Each student's interaction status in each NE was coded according to whether his or her utterances were playing the role of initiating a topic, responding to another group member, evaluating the content, or non-interactive. Importantly, the four categories were not ordered on a scale to suggest a hierarchy, but rather they were devised as descriptive categories. In our analysis, no more than one category was identified for each student in each NE.

Taking NE4 (see Table 1) as an example used to illustrate how we identified students' interactive patterns in the group discussion. As can be seen, during this NE, S4 initiated a discussion which had been addressed by the group; her first utterance was then be coded as initiation. S4's initiation was responded to by S2 who challenged the idea (S2 [4.1]) and provided a reason for his standing (S2 [4.2]). The following sentences

from S2 were then be coded as response. S3 followed S4's initiation and S2's challenge by saying, "...even a toilet bowl is not enough." S1 asked, "How is it impossible?" Combining with S1's following utterance (S1[NE4.8]), he was trying to evaluate whether S4's and S2's discussion was realistic in the problem context, i.e. the apartment, based on his own experiences and knowledge. His utterances were therefore coded as evaluation. Everybody was engaged in the discussion; therefore, no utterances were coded as non-interactive.

3) Narrative construction

To construct narratives, the researchers first created a chart to visualize students' interactive pattern in each NE during the whole group discussion. This helped to identify the characteristics of each student's participation during the activity. Narratives were constructed by referring to the chart; they were validated by reviewing the video recording and transcript. When constructing narratives, we mainly focused on the characteristics of each student's participation in terms of how they interacted with other group members during the whole group discussion and how the interactive pattern was related to the discussed topic. These narratives described, in detail, and allowed us to construct what position that each student took place. For example, if one student kept evaluating other students' ideas and maintaining the position of evaluator over a sequence of NEs, a narrative of evaluation might have been used to describe the details of how the student played his role as an evaluator and how other group members accepted or rejected the comment from this person.

IV. RESULTS

We first presented an overview of the sequences of NEs during the group discussion to give a sense of how the group solved the problem over the course of the discussion of various topics. We then zoomed into each NE by examining the shift of topics during the group discussion which led to our examination of the topic proposers in each NE (RQ 2). The next step was to examine students' interactive pattern along with the sequences of NEs (RQ 1). Finally, we provided contextual interpretations to students' positions through the three constructed narratives (RQ 1 and RQ 2) that linked students' interactive pattern with the topics the group discussed and the process of task completion within the group.

1. An overview of the sequence of NEs

Twenty-one NEs were identified based on the different topics. The sequence of NEs provided an overview of how the group completed the task and what topics the group talked about during the process. *Table 2* lists the person who initiated the topic discussion, the duration, and the topic(s) of each NE. Three NEs (NE3, NE14, and NE15) were not identified with any specific topic, during which students were mostly

Table 2 *Negotiative events (NEs) sequence and details*

Event number	Topic Initiator	Duration (m:ss.ms)	Topics
NE1	S3	0:23.92	How big is 60 m ² ? How many rooms are in the apartment?
NE2	S2	1:00.15	What kind of rooms could be in the apartment?
NE3	N/A	0:16.24	[Casual talk without a specific topic.]
NE4	S4	0:24.48	Is 2 m ² is big enough for a toilet?
NE5	S4	0:54.87	Is 5 m ² is big enough for a toilet? How big are four bricks?
NE6	S3	1:00.18	How long is a side of a brick?
NE7	S3	1:55.05	How big is one brick and how big are four bricks?
NE8	S4	0:10.91	Is 5 m ² is big enough for a toilet?
NE9	S2	0:31.24	Is a draft paper needed now for drawing the plan?
NE10	S3	0:59.51	Where to place the bathroom and toilet?
NE11	S4	1:10.19	How big should the other rooms (bedroom and study room) be and how many square meters are left?
NE12	S1	0:22.71	Should the group start to draw down the work?
NE13	S4	0:22.31	Is a rectangle of 6 × 10 suitable?
NE14	S1	0:23.08	[Students passing ruler with casual talk.]
NE15	S2	0:16.36	[Students passing rulers and erasers with casual talk.]
NE16	S1	0:57.43	Is a scale needed? How big is the living room?
NE17	S3	0:16.23	Is a rectangle suitable as the shape of an apartment?
NE18	S4	1:00.11	How many square meters are left for other rooms? For a study room, is the area okay? Are doors needed for drawing the work?
NE19	S4	0:43.93	Is 16 m ² big enough for a bedroom? Where should the living room be?
NE20	S3	0:38.66	What furniture should be in the living room? How many square meters are left for the kitchen?
NE21	S2	1:44.98	Is the plan possible in practice? [Labeling rooms and doors.]

Note: Descriptions in square bracket [] summarize the group actions or behaviors if no discussion topics were identified during that NE.

casually talking and passing writing tools. For NE14 and NE15, since the activities were related to the task completion process (passing writing tools), the person who initiated the activities was identified as the initiator.

Most of the time, the group was working on specific topics related to specific problem solving for completing the task, except for one NE (NE3) during which the four students were not trying to talk and respond to each other but rather they spoke on their own. The sequence of NEs was used to examine the process of task completion within the group. A categorization of the NEs in terms of what purposes they served for completing the task is described as following: The group discussion evolved in the following stages: 1) Clarifying the problem (NE1, NE2); 2) Discussing the basic plan of one room and its area (NE4, NE5, NE8); 3) Discussing the scale (the area of a floor tile) for estimating the area (NE5, NE6, NE7); 4) Discussing who could use what drawing tools (NE9, NE12, NE14, NE15); 5) Discussing a rough plan of the apartment (NE10, NE11, NE12); 6) Identifying the shape of the apartment (NE13, NE17); and 7) Discussing details regarding the area and location of each room (NE18, NE19, NE20). These NEs formed the basis of examining what topics the student talked about during the activities and the step-by-step process by

which the group completed the task.

2. Topic initiator and the shift of topics

With regard to each student's contributions to each NE (RQ 2), we recognized the topic initiator and how students shifted the topics during the discussions. Students proposed a different number of NEs during group discussions. Three, four, six, and seven NEs were proposed by S1, S2, S3, and S4, respectively. S3 and S4 initiated more topics than S1 and S2. The topics were shifted during the group discussion in a non-linear manner. Students visited and revisited some of the topics and each of the students were involved differently in the revisiting process. Three topics were revisited and each of the revisiting processes contributed to task completion.

The first revisiting process seemed to connect the group discussion about one room (toilet) and the discussion of the scale for estimating the area. The solution from the discussion of scale also solved the former problem. The topic initially proposed in NE5 was revisited again in NE8. The group first posed the question of how big a toilet should be (S4 initiated the topic), as the group did not seem to have a clear sense connecting "2 m²" to how big it is in real life. Then they worked on developing a common scale for estimating the size of the room using a classroom floor tile as a reference. After that, the actual question of, "How big should a toilet be?" was revisited and solved with reference to the area of a floor tile. For the revisiting process from NE5 to NE8, S4 played the vital role in driving the group to revisit the topic. She proposed the topic in NE5 and revisited it again in NE8, during which she led the mathematics calculation of the area with other group members until the problem was solved.

During the second revisiting process, the group started working on drawing rather than solely discussing the problem. The second revisiting of a topic happened in NE12 when the group revisited the original topic from NE9. During NE9 (initiated by S2), S2 suggested drawing the plan of the apartment on draft paper; this was rejected by the other students. However, the idea of drawing on draft paper was picked up by S1 later in NE12, and this time, the group accepted the suggestion. After a short discussion the group assigned drawing to S4. The revisiting process was conducted by S2 and S1, through which the group status transitioned from solely discussing to both drawing and discussing.

The group seemed to seek a shared agreement through the third revisiting process. In NE13, S4 asked the group whether a rectangle could be the possible shape of the apartment, but her question was not responded to by others at that time. After the group discussed drawing tools and organization of the physical drawing activities, S3 revisited S4's question of whether the overall shape of the apartment could be a rectangle and suggested that it might be difficult to allocate other rooms in the apartment if the apartment's shape was indeed rectangular. At that time, S1 was looking at her and seemed like he was thinking about S3's suggestions. S2 did not try to respond to S3; S4 looked at S3 and stopped her drawing. S3 did not receive further responses or evaluations from the group. S3 soon rejected her own suggestion

and said that “...an irregular shape would be difficult to draw.” This time, S1 and S4 nodded their heads and showed their agreement on using a rectangle. S4 then drew a rectangular-shaped apartment. S3 and S4 drove the revisiting of this topic and S3’s revisiting facilitated a shared agreement within the group (among herself, S1, and S4) to use a rectangle.

To complete a task collaboratively, a sequence of NEs were discussed by the students including topics about areas, shapes, functions of rooms, areas of each room, and design plans of the apartment. During these discussions, through revisiting some NEs, the group was able to connect the discussed content, organize group work (i.e., from discussing to drawing), or establish a shared agreement. Both initiating and revisiting topics were considered as ways to be involved in the discussions of different topics.

3. Students’ interactive pattern

Students’ interactive patterns in terms of their interaction category in every NE were identified (RQ 1). Table 3 summarizes how each of the students interacted with other group members in terms of the occurrence of different interaction category for the duration of the task. As can be seen from Table 3, S1 tended to evaluate information for the group; S2 kept his place among initiation and response and was non-interactive when it came time to evaluate information. S3 initiated more often than other members. S4 seemed to interact in various ways while evaluating more often.

Figure 2 provides a visual representation of how the students shifted among these categories along with

Table 3 Occurrence of interaction category for each group member

	S1	S2	S3	S4
Evaluation	9	0	1	8
Response	6	6	7	5
Initiation	2	5	10	4
Non-interactive	1	9	2	3

Figure 2 Students’ interactional patterns

the sequence of NEs (RQ 1). The horizontal axis represents the sequence of NEs chronologically. Students' interaction category in each NE are represented by the dots in different colors. S3 and S4 alternatively evaluated, responded, and initiated during NE4, NE7, NE9, NE10, NE11, and NE13, as well as from NE15 to NE20. In the first several NEs, S2 either initiated or responded, while after NE9, he only initiated once and maintained non-interactive till almost the end of group discussion. S1 evaluated more often than either initiation or response. S4 frequently shifted her participation between the four categories from the beginning until the end of the group discussion.

4. Narratives

Narratives were constructed to give contextual interpretation of how the four students interacted with each other (RQ1 and RQ2). We examined the connections among each student's interactive pattern, the discussed topic, and other students' interactive patterns (Figure 2). From this we constructed three narratives. The first narrative was about S3 and S4: We described the two students' participation as the narrative of an interactive pair. The pair was seen as taking up the position of leading the mathematics problem solving among the group. The second narrative was about S2: He shifted from interactive to non-interactive during group discussions. He was considered as a follower in CMPS. The last one was about S1: This was a narrative of evaluation that was constructed to describe how S1 kept his position as an evaluator during the discussions.

1) The narrative of the leading interactive pair-S3 and S4

S3 and S4 interacted in a coordinated pattern of initiation-response (evaluation) during multiple NEs (NE4, NE7, NE9, NE10, NE11, and NE13). The pair also interacted frequently and responded to each other closely from NE15 to NE20. During that period, one of the two students was initiating and the other was either evaluating or responding when drawing the apartment on the worksheet. The following gives an example

Table 4 NE19 (Topic: Is 16 m² big enough for a bedroom?
Where should the living room be?)

S4 (NE19.1):	Is the bedroom 16 m ² ?
S3 (NE19.2):	There is a living room still left over.
S4 (NE19.3):	How big should the bedroom be?
S3 (NE19.4):	Correct! There is a living room left over.
S4 (NE19.5):	Oh my god, it is terrible. Can the bedroom be near the study?
S3 (NE19.6):	Sure, just don't have the toilet opposite the kitchen.
S4 (NE19.7):	So the bedroom is here and the kitchen is here.
S3 (NE19.8):	Oh, one more room, and the living room.
S4 (NE19.9):	So the bedroom is 16 m ² ?
S3 (NE19.10):	That's ok, just do it [16 m ²], it's fine.

of the exchange between S3 and S4 during NE19 (Table 4). S4 initiated the discussion with her question, “How big should a bedroom be?” The whole NE consisted of S3 and S4 talking with each other without involving other group members.

In the beginning of the NE19 negotiation process, the two students each had their own focus (i.e., bedroom in NE19.1 and living room in NE19.2). S4 initiated the conversation by asking about the area of the bedroom. Even though S3 responded to S4, she did not answer S4’s question. S4 then asked again about the area of the bedroom, and this time, S3 quickly responded with “correct” while she again addressed her own focus on the leftover living room which had not been allocated in the apartment. As the discussion continued, S4 acknowledged S3’s comment regarding the leftover living room, saying, “that’s terrible.” Then S4 discussed the arrangement of the bedroom with S3; she also discussed the study and the living room. Toward the end of the NE, S4 asked again about the area of the bedroom, and this time, S3 responded by saying, “...Just make it [16 m²],” which led S4 to draw the room on the worksheet.

Rather than insisting on their own points of focus, the negotiation between S3 and S4 during NE19 shows that even though the two students began the discussion with different foci, they continued the discussion by responding to each other and negotiating in a similar manner to NE19 from NE15 to NE20. During the whole group discussion, the two students (S1 was sometimes also involved in the discussion) worked out the arrangement of rooms in the apartment and the area of each room before drawing the final solution on the worksheet. Their negotiation with each other constituted, in part, their participation during the collaborative task in mathematics, which also led the group completed the task. The pair’s role during the collaborative task was then considered as a leading pair of CMPS.

2) The narrative of the follower-S2

S2 shifted his participation from initiation to non-interactive during the group discussion, which appeared to be a significant shift. In the beginning of the group discussion, he showed a tendency to initiate discussion and respond to others. During NE3 and NE4, when the group was talking about the area of the toilet, S2 responded to S4’s idea of “a 2 square meters toilet” by saying “that was impossible, listen...” (S2, NE4.2). However, the sentence was interrupted by S3 who said, “That’s possible. 2 m²... uh, may be impossible. If we have 2 square meters, (the area) is even not enough for a toilet bowl.” S3’s utterances suggested that she changed her mind about S2’s suggestion midway. S3 actually interrupted and rejected S2 as soon as S2 was speaking, but then she realized that S2’s thinking of “2 square meters is not enough” was reasonable. As the discussion continued, S4 proposed her new idea of planning a toilet of 5 square meters. S2 was still trying to give some explanations as to why 2 square meters was not enough for a toilet and what a toilet should look like, but he did not receive any responses from the other group members.

The group then started to discuss whether 5 square meters was enough for a toilet and they worked to estimate the size of one square meter by using the area of a classroom floor tile. During this period, until the area of a floor tile and the area of the toilet had been determined (NE5, NE6, NE7, NE8), S2

approached the interactions several times by responding to others without providing suggestions or giving explanations for his statements (e.g., “[S4’s name], it really doesn’t have that.” [NE6.13]). His initiations of discussion in NE6 and NE8 were all responded to and interrupted by S3.

When S2 was about to design and draw on the worksheet, S3 soon interrupted S2, saying, “No, wait...” S3 continued her interjection by proposing her own questions; other group members (S1 and S4) also responded to S3 (i.e., NE6.3, NE6.5) rather than S2. Similar rejections happened in NE8 and NE9.

S2’s interactions with other group members started with actively proposing ideas or responding to others. However, the consecutive rejections of his participation in NE6, NE8, and NE9 and being ignored by other students in NE5 and NE7 seemed to make him to engage in self-talk (NE10, NE11, NE13, NE15, NE16, NE18, and NE20) rather than interacting with other group members. As seen from his interactions with other group members, his participation approach could be described as starting from initiating topics, turning to following other group members’ ideas and then finally engaging in his own self-talk. His shift from initiation or response to a non-interactive position showed a diminished level of participation. His engagement in self-talk in the later stage of group discussion indicated that he to some extents failed to follow the problem solving process led by other group members.

3) The narrative of the evaluator–S1

S1 played his part mostly as an evaluator. He rarely initiated conversation or directly responded to the initiation person; he also never engaged in self-talk. In the beginning of the discussions from NE1 to NE6, S1’s interactions were not very frequent. He mostly observed other group members’ discussions. Sometimes he responded to others or proposed questions about other students’ discussion content. In NE6, when S3 and S4 were discussing the area of one floor tile, S1 asked, “How could 100 square decimeters equal to 1 square meter?” (S1[6.8]). In NE7, S1 asked, “60 multiplied by 60, how could it be 360?” (S1[7.8]). His questions were all addressed by other group members and arose discussions about mathematical content during the task completion process.

In NE12, S3 asked S1 to draw the plan on the worksheet; this was rejected by S1. S3 then asked S4 to draw; S4 began drawing. During the drawing process, S1 helped organize and pass physical drawing tools (i.e. rulers, papers, etc.) to S4. As illustrated earlier, from NE13 to NE18, S3 and S4 interacted quite often with each other. During this time S1 sometimes responded to their discussion and echoed their designs (NE13.3); he sometimes gave more information and instructions (NE18.10); and he sometimes organized the passing of drawing tools among group members (NE14.2).

S1’s utterances were not as frequent as S3’s and S4’s, but most of them were responded by the group during problem solving activities. Through playing his part as an evaluator, his suggestions and questions were heard and addressed, he also helped the pair S3 and S4 to clarify problems and solving mathematics calculations during the task completion process. His relatively frequent occupation in the evaluator position and the way his questions were addressed by the group suggests the indispensable role of

evaluator during completing the collaborative task in mathematics.

V. DISCUSSION AND CONCLUSION

The examination of students' interactional patterns, namely initiation, response, evaluation, and non-interactive, provides a way of understanding how students interact with each other, in terms of who is talking to whom during the task completion activity. With regard to the four interaction categories, students appeared to show individual characteristics in terms of which categories they frequently took in group discussion. These interactional patterns could be considered as part of student participation which gives different students various entries to collaborative task completion activities. This study was additionally designed to consider the content of group discussion. By focusing on the negotiation process and tracing the sequence of NEs, the topics of discussion and shared, intermediate goals were identified. Each student's contributions were studied, in terms of how he or she was involved. Through initiating or revisiting topics, students involved themselves differently during the group discussions. For instance, S3 and S4 often led the revisiting process, while S1 and S2 did not have much involvement with it.

In this study, the narratives constructed by the researchers describe how student interactions developed and evolved over time. In addition to the description of the moment-by-moment positions the students assumed, the use of narratives provides more contextual interpretation of how students interacted with each other. As suggested by the narratives, the content during the negotiation process might play a critical role in influencing students' interactional patterns. For example, S2's shift from an active interactive category (initiation or evaluation) to a non-interactive one might have been due to the consecutive rejections from other group members. While looking at the group discussion during these rejection moments, in most cases, S1, S3, and S4 were focused on the mathematical content, but S2's focus was on drawing and other social contexts related to the task. This reflects on previous work in which researchers found that certain content might encourage or inhibit students' involvement in group activities (e.g., Wood, 2013).

These results verify the possibility of looking at student participation by conceptualizing student participation as a process of taking part in student interaction and task completion, through taking different roles and involving in the discussion of different topics during collaborative problem solving activities. Theoretically, these findings build on the use of positioning theory and the examination of the negotiation process which led to our investigation of the relationship between positioning and negotiation. Different with previous literature where a possible unfair assessment of students inherent status could be referred to, this study addresses that student participation in collaborative task in mathematics is influenced by group dynamics and also the task completion process (Hare, 2003). Taking both students' interactional pattern and their involvement in the content of group discussion into consideration, we would be able to track each student's position they took in particular situations during task completion in groups. This also allows us to provide more direct and targeted instructions in terms of how to encourage

student participation in the process in practice.

Several issues arise from this study which are worthy of addressing. The first is regarding the mechanism of initiating and revisiting topics during group discussions. Previous researchers noted that during a collaborative task in mathematics, the process of accepting or rejecting a topic (i.e., mathematical topic) was complicated and significant for understanding student participation (Barron, 2003). In this study we looked at how each student initiated topics, how these topics were accepted or rejected within the group, and how some of the topics were revisited. Further questions based on this study might be related to why a specific topic was accepted, rejected, and/or revisited and who was involved in the process. In this group, the revisiting process was important in supporting the task completion. But the results here illuminate the need for more research on each of these.

The investigation of students' positions brings up further questions about the dynamics of participation during student interactions. Different with previous research that use the initiation-response-evaluation model to describe teacher-student verbal interactions during classroom teaching (Cazden, 2001; Dong et al., 2019; Mehan, 1979), this study explored a way to use the initiation-response-evaluation model to examine students' interactions through their verbal actions. The results show the influence of other students' on each student's interactive way during group discussion. The results also suggest that students take certain ways to interact with other group members have more privilege in terms of controlling the group discussion and participating in the task completion process. This contributes to understanding the social constructions of power relations among students through the lens of positions (Esmonde, 2009; Langer-Osuna, 2016; etc.). For example, although S2 tried to initiate conversation in NE6 and NE8, S3 and S4 responded to S2 but controlled the group discussion during the two NEs. S3 and S4 seemed to participate more actively in the task completion process through establishing stable interactive relationships by consistently initiate-respond-evaluate. Future research is needed to investigate the how such relationships among the students were shaped and how certain position(s) might support students in controlling the group discussion.

These findings support the conceptualization of student participation through positioning and negotiation. The study also contributes to our understanding of student interactions and the content of group discussions during a collaborative task in mathematics. It is always difficult to determine student participation during group activities by taking only one perspective into consideration (Stasser and Vaughan, 1996). The significant advantage of this conceptualization of student participation may be that multiple perspectives are taken into consideration. This provides various ways for teachers to look at student participation and talk about student behaviors in teaching practices so as to improve their instruction and help to improve students' skills in participating in a collaborative task in mathematics. In order to give a more comprehensive picture, more groups need to be studied. Based on the Social Essentials of Learning project, a comparative study between students from Australia and China might also be interesting in terms of looking at the cultural factors that may influence student participation in

different countries. This study serves as a starting point for conceptualizing student participation and understanding the dynamics of collaborative task completion in mathematics. The integration of different theoretical lenses provides an operational approach of analysis in this study, while also raises further questions and concerns about the theoretical meaning in terms of the social and cognitive relations behind positioning and negotiation.

CONFLICTS OF INTEREST

No potential conflict of interest relevant to this article was reported.

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