Original Article

Preparing Students for the Fourth Industrial Revolution through Mathematical Learning: The Constructivist Learning Design

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Abstract

Characterised by increased automation and digitalisation of work processes, the Fourth Industrial Revolution (4IR) has displaced and redesigned many existing jobs, and will create new occupations that are currently non-existent. To prepare a future workforce that is adaptive amid a volatile employment landscape, schools should provide the necessary learning experiences to help students today develop transferrable competencies, which encompass deep conceptual understanding of domain-specific knowledge and 21st century competencies in the cognitive, intrapersonal, and interpersonal domains. In this paper, we study this possibility in the context of mathematics learning and propose a constructivist learning design (CLD) that affords students to engage in deeper learning processes. In the proposed CLD, students first work collaboratively to solve a complex problem targeting a math concept that they have yet to learn, before being engaged in instruction that builds upon their solutions in the teaching of the concept, and practices that reinforce these ideas. Testing CLD in mathematics learning at secondary level via a quasi-experimental design, we found out that (1) CLD facilitates deeper learning as it encouraged students to apply their cognitive, intrapersonal, and interpersonal competencies, and (2) CLD students (n=23) outperformed their Direct Instruction counterparts (n=18) on mathematical conceptual understanding and transfer. Overall, this study suggests that the CLD has the potential to cultivate competencies that allow students to transfer in novel situations, rendering it as a possible learning environment to better prepare students for the 4IR.

Keywords: constructivist learning design, fourth industrial revolution, mathematics education, transfer, twenty-first century competencies

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I. INTRODUCTION

The Fourth Industrial Revolution (4IR), which is characterised by dramatic technological advancement and increased globalisation, heralds a profound transformation of labour markets (World Economic Forum [WEF], 2020). A research conducted by the McKinsey Global Institute (Manyika et al., 2017) showed that automation technologies, which include artificial intelligence and robotics, will likely displace on average 15 percent of jobs in 46 countries by 2030, with the impact felt more acutely by advanced economies. With technological progress inadvertently resulting the creation of occupations that do not exist today, or redesigning work in existing industries (e.g., health care, energy), millions of people may need to switch occupations or upgrade their skills by 2030 (Manyika et al., 2017). Given this volatile, uncertain, complex, and ambiguous (VUCA) 4IR environment, one's future employability will depend largely on whether he or she can respond effectively as "valuable knowledge workers" (Brynjolfsson & McAfee, 2014) and be adaptable as their occupations evolve alongside capable machines (Lewis, 2020). Apart from higher educational attainment, which could include sound foundational knowledge in science, technology, engineering, and mathematics, the desired adaptive worker has skills that are hard to automate, like high-level cognitive capabilities, socio-emotional skills, creativity, complex communication (Brynjolfsson & McAfee, 2014; Gleason, 2018; Manyika et al., 2017), and the ability to unlearn, learn, and relearn (Ra et al., 2019). Given that workers of the future need to possess these skills and knowledge to nimbly transfer their existing skills and knowledge to adapt in an everchanging employment landscape, how can schools help students nurture these competencies in order to prepare them for a challenging and volatile 4IR environment?

We examine this question in the context of mathematics education, considering mathematics' underpinnings in the science, technology, and engineering disciplines, and its continued presence in powering a digitalised, data-driven, and globalised environment (Maass et al., 2019; Organisation for Economic Co-operation and Development [OECD], 2018). However, given the demands of the 4IR environment, there is a need for us to re-evaluate the kinds of competencies that students should develop in the mathematics classroom. With automation and computers taking away much of computational work, mathematics education should move students beyond the mastery of mathematical procedures and help them to develop a deep conceptual understanding of mathematics. Students with deep conceptual understanding will be able to grasp the mathematics underlying the programming of machines, generate and apply mathematical models to interpret, explain, and make predictions during problem solving, and evaluate the plausibility of mathematical results (Gravemeijer et al., 2017). Such conceptual understanding, which sees a more connected knowledge of the meaning and structure of mathematics and the relationships among mathematics concepts, is one that facilitates flexible transfer and generalisation of mathematics knowledge in novel problem contexts (Richland et al., 2012; Skemp, 1976).

Merely concentrating on developing students' mathematical conceptual understanding to prepare

them for the challenges in the 4IR environment will however be inadequate. A diverse range of skills or competencies, collectively dubbed as 21st century competencies that include but are not limited to cognitive and social-emotional competencies (or “soft” skills) like problem solving, communication skills, creativity, collaboration, critical thinking, decision making, and self-direction, are also identified by many researchers as important for the 4IR (see Chaka, 2020 for a scoping review). These affective, dispositional, and volitional competencies will be necessary when working with complex, messy work situations that may have unknown or many solutions, and that require flexible and adaptive problem-solving skills.

Taken together, learning environments in mathematics classrooms need to help students develop both hard competencies that allow for the deep understanding of mathematics and soft, 21st century competencies, in preparation for students to develop the skills and knowledge to navigate in the VUCA 4IR environment. In our search for possible competency frameworks that could inform us on how such environments can be designed, we leveraged the one proposed by the “US Committee on Defining Deeper Learning and 21st Century Skills” (Pellegrino & Hilton, 2012), where 21st century competencies are defined as *transferable skills and knowledge* “that are specific to – and intertwined with – knowledge within a particular domain of content and performance” (p. 3). Three broad domains of 21st century competencies – cognitive, intrapersonal, and interpersonal – were identified, and these resonated with the hard and soft skills necessary for the 4IR and the 21st century workplace (Chaka, 2020; Gravemeijer et al., 2017; Ra et al., 2019; WEF, 2020). Adapting from the framework, we argue that the competencies that are important to be developed in the mathematics classrooms include the following:

- *Cognitive domain* includes thinking, reasoning, and disciplinary content skills. Reviewing the competencies for the 4IR (e.g., WEF, 2020) and related mathematics competencies (e.g., Niss & Højgaard 2019), we identified complex mathematical problem handling/modelling, mathematical thinking and reasoning, inventiveness, and mathematical conceptual understanding as important cognitive competencies.
- *Intrapersonal domain* includes the ability to regulate one’s behaviours and emotions to attain certain goals. Surveying the relevant competencies from research in 4IR (e.g., WEF, 2020), we identified persistence and self-direction as focal intrapersonal competencies.
- *Interpersonal domain* includes expressing information to others, and interpreting information from others. From the relevant competencies from research in 4IR (e.g., WEF, 2020), these will include communication and collaboration with others.

The definitions of the various competencies can be found in *Appendix Table 1* of the Appendix. For the development of the aforementioned competencies, learning environments that afford *deeper learning*, defined as the “process through which an individual becomes capable of taking what was learned in one situation and applying it to new situations (i.e., transfer)” (Pellegrino & Hilton, 2012, p. 5), are needed.

Despite the increasing attention on the necessity of cultivating deeper learning for the 21st century learner, evidence of how these competencies could be modelled, cultivated, and assessed remains limited (e.g., Conley & Darling-Hammond, 2013). In a recent meta-review conducted by Sergis and Sampson (2019), the authors pointed out that most of the 38 identified research studies addressed some but not all competencies. In this paper, we propose an instructional design, coined *constructivist learning design*, that may address this gap and afford students opportunities to engage into all domains of deeper learning. In the explication of this instructional design, we (1) examine the learning and instructional principles under the deeper learning framework, (2) demonstrate the theoretical alignment between deeper learning and constructivist learning theories, (3) show how this instructional design which embodies constructivist learning principles may afford students opportunities to engage in deeper learning, and (4) examine the efficacy of the constructivist learning design, against the conventional direct instruction, in engendering the desired competencies for the mathematics classroom that may prepare students for a 4IR future.

II. LITERATURE REVIEW

1. Deeper learning for the mathematics classroom

In preparing students to be successful in solving new problems and adapting to novel situations, Pellegrino and Hilton (2012) argued for *deeper learning*, which involves the application of cognitive, intrapersonal, and interpersonal competencies to novel problem situations. This process in turn help to develop these competencies, forming a recursive cycle of application and development. At the heart of deeper learning is the development of transferable skills and knowledge, which involves having a strong content knowledge of a domain (e.g., mathematics), and the understanding of when, how, and why to apply the content knowledge in new situations for the purpose of both problem solving and new learning (Goldman & Pellegrino, 2015; Pellegrino & Hilton, 2012).

Drawing from cognitive psychological perspectives, Pellegrino and Hilton (2012) noted that transfer within a subject area or domain is possible with (i) effective instructional methods, (ii) well-organised knowledge, which includes integrated facts, concepts, procedures, and strategies that can be readily retrieved to apply to new problems, (iii) extensive practice that is aided by explanatory feedback that can help learners correct errors and practice correct procedures, and (iv) meaningful learning, which involves the understanding of the structure of the problem and the solution method. However, beyond the cognitive competencies, deeper learning and transfer should be supported by intrapersonal competencies that reside within an individual and operate across a variety of life contexts, and these include intellectual openness (e.g., flexibility, adaptability, and social responsibility), work ethic (e.g., self-direction, and perseverance), and self-evaluation (e.g., self-regulation). In addition, citing studies on the importance of social context in learning and social processes underlying self-regulated learning through help seeking and collaboration,

Pellegrino and Hilton (2012) noted the importance of social and interpersonal skills in supporting deeper learning that transfers to new classes and problems, and in enhancing academic achievement.

To effect deeper learning in a learning design, the design's environment should mimic the sort of learning demands that are dictated by the workplace of the future. Given the volatile nature of future workplaces, where change is the only constant, it is possible that students will engage in the processes of learning, unlearning, and relearning (see Hislop et al., 2014; Peschl, 2019; Sharma & Lenka, 2019 for discussions of these constructs in work settings). Hence, the learning design should allow students to experience such processes, allowing them to work with what they already know and build upon these in the acquisition of new knowledge. Such considerations harmonise with the constructivist theoretical orientation to learning, which could suggest the necessary mechanisms that would invoke the deeper learning process to effect transfer.

2. Constructivist orientation for deeper learning

As a metaphor for learning, constructivism proposes that knowledge is a product of our own cognitive acts (Confrey, 1990a) and is actively constructed by us (Karagiorgi & Symeou, 2005; Noddings, 1990). Compared to two other predominant theoretical positions - behaviourism and cognitivism - on learning, the constructivist position neither subscribes to the mind-independent nature of knowledge (Ertmer & Newby, 2013) nor believe that knowledge could be mapped, imposed, or transferred intact from the mind of one knower to another (Applefield et al., 2001; Ertmer & Newby, 2013; Karagiorgi & Symeou, 2005). Knowing, from a constructivist position, is an adaptive process, and knowledge that is constructed must be viable for, or make sense to, its agent under the particular circumstance in which learning takes place (Karagiorgi & Symeou, 2005).

Given that learners construct individual interpretations of their personal experiences, the constructivist position posits that learners' conceptions of knowledge are derived from a meaning-making process (Applefield et al., 2001). However, learning typically takes place in a social context, and hence individual constructions can be influenced by the power of interaction and negotiation (Jaworski, 1994). As pointed out by Vygotsky (1978), the process of learning is done in collaboration with the teacher in instruction (Green & Gredler, 2002), and individuals make meaning through the interactions with each other and the environment that they live in (Amineh & Asl, 2015). As such, to understand how learners transfer what they learn from one setting to the other, this perspective points to the context or environment of learning, and how the learner interacts with the environment to acquire the knowledge that is embedded in it (Billing, 2007; Ertmer & Newby, 2013).

Drawing from these constructivist perspectives, we identified three proposals from constructivist learning principles that may shed light into the deeper learning process. First, *understanding is brought about through an interaction between learners' prior conceptions and the context of learning*. This proposition

emphasises the importance of a learner's prior knowledge in the acquisition of new knowledge, and in the transfer of learned knowledge to novel contexts (Billing, 2007). When faced with a novel task or concept, learners' prior conceptions, whether formal or informal, will be activated and used as "resources" in the knowledge construction process (Smith et al., 1994). Research on students' misconceptions and alternative conceptions (e.g., Confrey, 1990b) provided evidence of this as they could be viewed as adaptations of the knowledge construction process. Since understanding is an individual construction, the learning environment must possess features to allow for the compatibility of these individual constructions to be tested (Savery & Duffy, 1995), and these could be achieved through reflections and comparisons of current practices (Billing, 2007).

A second theme gathered from constructivist views on learning is that *learning is stimulated via cognitive conflict or disequilibrium, which determines the organisation and nature of what is learnt*. Learning is defined by its goal, and the goal defines what the learners attend to and the prior experiences that h/she brings (Savery & Duffy, 1995). What is "problematic" leads to, and is the organiser for, learning (Dewey, 1938; Roschelle, 1992). In line with Piaget's (1970, 1977) theory of cognitive development, knowledge construction is stimulated by internal cognitive conflict as learners strive to resolve mental disequilibrium (Applefield et al., 2001). Whether the learning of the targeted concept occurs next depends on whether the concept is assimilated or accommodated into students' schemas. Assimilation occurs when students try to fit new information into their existing schemes, whereas accommodation occurs when the target learning conflicts with existing schema, prompting reorganisation of one's schema structure.

Getting learners to realise the potential between their current knowledge and that of the targeted one also harmonises with Vygotsky's (1978) "zone of proximal development" (ZPD), where this "zone" illustrates the difference between what a student could achieve independently and what he or she could achieve with the guidance of knowledgeable others (e.g., teachers, peers). Teachers could help students to realise their potential and acquire more complex skills via several instructional means, such as through problem situations that result in impasses (VanLehn et al., 2003), failure (Kapur, 2008, 2010), ambiguity (Voigt, 1994; Foster, 2011), or uncertainty (Zaslavsky, 2005). These experiences not only compel students to surface their current understanding to make sense of the uncertain situation, but also enable teachers to build on students' ideas, strategies, and preconceptions in their instruction. This would help to close the ZPD by linking students' current understanding to the targeted concept to be learned.

The third notion that constructivism could shed light on deeper learning is *the importance of evaluating the viability of individual understandings and social negotiation in the evolution of knowledge*. Dialogue is the catalyst for knowledge acquisition, and understanding is facilitated by exchanges that occur through social interaction, questioning and explaining, challenging, and offering timely support and feedback (Applefield et al., 2001). Knowledge is mutually built when learners both refine their own meanings, and help others find meaning; it is also enabled via the supportive guidance of mentors as they enable the apprentice learner to achieve successively more complex skill, understanding, and ultimately independent

competence (Applefield et al., 2001). Meaning can be socially negotiated and understood based on viability (Savery & Duffy, 1995).

Transfer is promoted when learning takes place through active engagement in social practices that embed its understanding; it is also facilitated when learners are encouraged to talk about the similarity of representations for both the initial and targeted tasks (Billing, 2007). Complex problem-solving activities with peers might be appropriate platforms to develop persistence and generative learning strategies. The development of effective learning strategies and knowledge of when to use them can be modelled by teachers, who encourage self-regulated learning in the process (Applefield et al., 2001).

The constructivist propositions outlined above could suggest a set of instructional principles that could help achieve deeper learning of transferable skills and knowledge (i.e., 21st century competencies). Building on the key learning principles that were outlined by Goldman and Pellegrino (2015) on deeper learning, we posit that the instructional design should

- *afford the elicitation and building upon of students' pre-existing understandings of a subject matter.* These pre-conceptions could comprise both formal and informal knowledge that students have of a concept or topic. The elicitation could be realised through a complex problem that contains the stimulus that afford the students to model various features of a concept or topic through the use of variation (Dienes, 1960; Marton, 2006) and generate their various initial conceptions;
- *aid in the development of an organised and interconnected knowledge that facilitate retrieval and application.* The preconceived notions of the concept or topic that is targeted in the task will be built upon during instruction, and their viability being evaluated against the critical features underlying the targeted concept or topic;
- *engage students' thinking about their thinking and learning* through cognitive disequilibrium, the realisation of one's potential and reflecting on the affordances and constraints of their solutions. These help to cultivate critical thinking, and self-directed learning; and
- *build a social surround that allows for interpersonal and social nature of learning.* This could take the forms of collaborative learning and the orchestration of socio-mathematical norms by the teacher to allow the negotiation of meaning of the targeted concept.

3. Constructivist learning design for the mathematics classroom

Drawing from the specifications from the instructional principles proposed for deeper learning that is based on constructivist principles, a two-phased “problem-solving first, instruction later” instructional design (Loibl et al., 2017) was proposed to cultivate the development of competencies that are necessary for the 4IR. Coined *constructivist learning design* (CLD), it comprises 2 phases, (i) a *problem-solving phase*, where students work in groups to solve a complex, open-ended problem targeting a concept that

they have yet to learn and (ii) an instruction phase where the teacher builds upon the student-generated solutions from the problem-solving phase to teach the targeted concept. To reinforce the connections and linkages that were built during the instruction phase, students will also work on practice questions that are calibrated to the ideas and critical features that were discussed during the instruction.

1) Problem-solving phase

The problem that is used in the collaborative problem-solving phase is designed such that it helps to elicit the prior knowledge structures of students. The complex, open-ended nature of the problem encourages students to actively discover as many solutions as possible, giving them the opportunity to tap on their intuitive or formal prior knowledge when engaging in a problem situation. In line with similar “problem-solving first, instruction later” designs (Loibl et al., 2017) like the Japanese Open-Ended Approach (Becker & Shimada, 1997) and Productive Failure (Kapur & Bielaczyc, 2012), the problem is complex with various parameters for students to consider and open to multiple solutions as students attempt to find ways to solve the problem for which the targeted concept or strategy has not been taught. Past research suggests that while students will typically be unable to discover the correct solutions by themselves, they are able to generate a rich diversity of solutions (e.g., Kapur, 2008, 2010, 2012; Kapur & Bielaczyc, 2012).

Peer collaboration in this problem-solving phase not only allows for the negotiation of meaning among peers but is also important for the development of 21st century competencies, such as communication and collaboration skills. The teachers’ role in this phase is to ensure conceptual conflict and disequilibrium. Specifically, while students are engaged in problem solving, the teachers’ role is to facilitate students’ problem-solving efforts by pointing out the potentials of students’ solutions and suggest ways to refine their strategies, thereby inducing conceptual disequilibrium and prompting them to seek out other solutions. From the observations of students’ solutions, teachers could also identify the students’ ZPD (Vygotsky, 1978). In addition, it is also the teachers’ role to keep the groups on task and provide them with affective support to ensure that groups persevere in their problem-solving effort.

2) Instruction phase

Following the problem-solving phase, the teacher will take the solutions that were produced by his or her students and build on them to teach the targeted concept or strategy. The aim of the instruction phase is the resolution of the conceptual disequilibrium that was being induced during the problem-solving phase and effect the process of assimilation and accommodation (Piaget, 1977) in the understanding of why the targeted concept is the most viable given the problem. In line with the research from Productive Failure (Kapur, 2008, 2010), students’ solutions are organised according to their relationship with the critical features of the targeted concept, as a means to understand what the concept is and what the concept is not (Dienes, 1960). The affordances and constraints of each solution type are compared and

contrasted with the critical features of the targeted concept, through the use of counter examples as much as possible. Through getting students to analyse the viability of their solutions, the instruction phase becomes a platform for the negotiation and reflection of the meaning of the concept under study.

In line with the guidelines of the Chinese Post-Tea House approach (Tan, 2013), the deep conceptual understanding developed by a constructivist approach in teaching should be accompanied with tasks that not only allow students to reinforce the procedural understanding of the concept, but also to demonstrate and apply such deep understanding. Given that the instruction has allowed students to move beyond the procedural understanding of the concepts, teachers could design higher-order practice questions that show the affordances and constraints of concept in certain contexts. Hence, the CLD provides suggestions to the type of higher-order questions that teacher could leverage after getting students to achieve a connected understanding of the concept and cultivation of transfer.

3) Constructivist learning design and 21st century competencies

Past research and reviews have demonstrated that problem-centred and inquiry-based learning designs, which are similar to that of the CLD, could facilitate the development of learners' cognitive, intrapersonal, and interpersonal competencies. Reviews of problem-based learning (PBL) used in various levels and settings, such as K-12 classrooms, tertiary medical education, and professional training development (e.g., Hung et al., 2008; Merritt et al., 2017; Thomas, 2000), have shown that students who underwent PBL acquired better content and conceptual knowledge following the intervention. Research of similar two-phased "problem-solving first, instruction later" instructional designs (see Loibl et al., 2017 for a review) also demonstrated the potential of such designs in helping students transfer. For example, in the "Productive Failure" research in Singapore mathematics classrooms, it was found that Productive Failure students significantly outperformed their counterparts in the traditional direct instruction (DI) condition on conceptual understanding and transfer problems without compromising on procedural fluency (Kapur, 2008, 2010, 2012; Kapur & Bielaczyc, 2012). In terms of inventiveness, there is evidence from research on students in middle grades that showed the relationship between PBL curricula in the development of mathematical creativity (Chamberlin & Moon, 2005; Kwon et. al., 2006).

With regard to intrapersonal skills, a review by Hung and his colleagues (2019) noted that PBL students in the context of medical schools demonstrated greater self-directed learning skills (e.g., making greater use of library resources) than conventionally trained ones. As for interpersonal skills development, reviews conducted on PBL (e.g., Hung et al., 2019; Hung et al., 2008; Schmidt et al., 2009) have shown that PBL students, mainly in the medical education settings, demonstrated better communication and collaborative skills than their traditionally taught counterparts. In mathematics learning, a follow up assessment of five second-grade classes who underwent a problem-centred learning reported having stronger beliefs about collaboration compared to their traditionally instructed counterparts (Cobb et al., 1992).

Taken together, these findings suggest that such problem centred instructional designs could afford

various aspects of deeper learning. Nonetheless, as identified by a review done by Sergis and Sampson (2019), many studies that sought to understand how their learning designs effect deeper learning did not cover competencies of all domains (i.e., cognitive, intrapersonal, and interpersonal). Furthermore, we observed that many of the studies in the reviews of problem-centred learning designs (e.g., Sergis and Sampson, 2019; Hung et al., 2019) had employed self-report measures to assess intrapersonal and interpersonal competencies. While it may be useful to have direct measures on the intrapersonal and interpersonal competencies developed, these self-report measures present challenges in understanding the transferability of these competencies (Pellegrino & Hilton, 2012). These competencies may also be domain specific (Stecher & Hamilton, 2014), and adopting general measures of such competencies may be problematic. As such, we may have to rely on other means to tease instances of students' application of intrapersonal and interpersonal competencies, such as the researchers' observations of students' behaviour, and other process measures.

The CLD research aims to address these gaps, to see if a problem-centred learning design that embodies constructivist principles could afford the application of the deeper learning competencies, and in turn help students develop transfer. How the various constructivist principles and competencies fit in this two-phased design is shown in *Table 1*.

As shown in *Table 1*, the problem-solving phase involves the use of cognitive mathematical competencies like complex mathematical problem handling and modelling, with the generative efforts of

Table 1 *Features of the constructivist learning design with respect to its constructivist underpinnings and alignment to competencies (mathematical and 21st century competencies)*

Phase	Feature	Constructivist processes & underpinnings	Mathematical and 21 st century competencies applied
Problem solving phase	Complex, open-ended problem task	Activation of prior knowledge, ZPD and Impasse or failure to induce conceptual conflict	Cognitive competencies: complex mathematical problem handling, mathematical modelling, mathematical thinking and reasoning, inventiveness
	Peer collaboration	Communication and negotiation	Intrapersonal: Persistence and self-direction Interpersonal: communication, collaboration with others
	Teacher facilitation	ZPD and conceptual conflict	Cognitive competencies: mathematical thinking and reasoning Interpersonal: communication
Instruction phase	Consolidation of solution methods	Conceptual conflict, negotiation of meaning, assimilation & accommodation	Cognitive competencies: mathematical thinking and reasoning, conceptual understanding Interpersonal: communication
	Instruction of procedures and practices	Scaffolding of procedural knowledge	
	High-order practice questions	Critical thinking	

students to come up with as many solutions as possible. The novel situation created also gives rise to the development of students' inventiveness in coming up with multiple solutions to solve the problem. Given the complex nature of the problem, this also helps to develop students' persistence and self-direction to formulate the solutions. Working in a groupwork setting, as well as the affective support provided by the teacher, afford students to cultivate the necessary collaborative and communication skills, compared to if they were to solve the problem independently. The teacher-led instruction phase, which involves the consolidation of students' ideas to teach the targeted concept, and the affordances and constraints of their ideas, is less evaluative compared to more transmissionist forms of instruction. Such discourse encourages a classroom culture that allows students to develop the necessary communication skills to express their thinking behind their solutions. The instruction phase further helps to hone the mathematical thinking and learning competencies to allow for a more connected understanding of the topic taught, and a deeper understanding of the mathematics.

4. The present study

The present study aims to establish the efficacy of the CLD in the actual ecologies of the mathematics classroom as a means of simulating deeper learning processes, which afford the development of transferable competencies that will prepare students for the 4IR. We will examine how the CLD affords students to apply cognitive, intra-, and interpersonal competencies during learning, and these are observed via relevant process measures (see the Methods section and *Appendix Table 1* of the Appendix for more details). As to how CLD could engender transfer, we examine these from learning outcomes that are mainly cognitive, and these include the deep understanding of mathematics (i.e., conceptual understanding), and the ability to use one's mathematical understanding to flexibly respond to novel and unfamiliar problem situations (i.e., ability to transfer). Based on past research in terms of cognitive competencies transferred, we hypothesise that:

- H1. Students who were taught using the CLD would not significantly differ from students who were taught using DI in procedural knowledge.
- H2. Students who were taught using the CLD would demonstrate higher levels of conceptual understanding as compared to students who were taught using DI.
- H3. Students who were taught using the CLD would demonstrate higher ability to transfer as compared to students who were taught using DI.

We acknowledge that learning outcomes could also be measured in non-cognitive terms (e.g., the development of communication skills or self-directed skills), but we note that the development of such intra- and interpersonal learning outcomes requires more extended interventions and may not be reliably

assessed in short interventions such as the one reported in this study. Nonetheless, we will attempt to measure and comment on any intrapersonal and interpersonal learning outcomes that arose from the implementation of the CLD unit.

III. METHODS

1. Participants

A total of 41 Secondary One students (seventh grade; ages between 12.5 and 13.5-year-old) from a mainstream secondary school in Singapore participated in the present study. The students were from two intact mathematics classes, with each class taught by a different teacher (n=2 male teachers). The targeted mathematical concepts taught was “gradient of linear graphs”, a topic that was part of the Secondary One mathematics syllabus (Ministry of Education, 2012). One of the classes was assigned the CLD condition (n=23; 8 females) while the other class proceeded with the conventional DI method of instructing the topic (n=18; 7 females; more details of the DI can be found in the “research design” section). Students in both classes were not taught the concept of gradient of linear graphs prior to the intervention study.

2. Research design

A pre-post quasi-experimental design, with the CLD condition and its DI counterpart, was employed for this study. In the CLD condition, students first underwent a problem-solving phase followed by an instruction phase. In the problem-solving phase, students worked collaboratively for 50 minutes to generate as many solutions as possible to a complex, open-ended problem targeting the concept of gradient of linear graphs that they had not been formally instructed. This problem-solving phase prepared them to learn from the instruction phase in which the teacher discussed the students’ solutions, compared and contrasted them, and in the process, brought out the canonical targeted concept and its critical features. The teacher then got students to practice problems in class and as homework, targeting the necessary procedural and conceptual understanding of the topic from the school’s prescribed textbook. In addition, to supplement the ideas that were brought up during the consolidation of ideas, additional questions were designed for practice.

The DI condition differed from its CLD counterpart in terms of the sequence of the problem-solving and instruction phases. DI students first experienced the teacher-led instruction guided by the course textbook. The teacher introduced the targeted concept to the class, scaffolded problem solving by modelling and working through some examples, encouraged students to ask questions, and then discussed the solutions with the class. After the instruction, student solved problems as practice in class and as homework, targeting the necessary procedural and conceptual understanding of the topic, from the same textbook as

that of their CLD counterparts. The teacher went through the solutions of the problems, directing attention to the critical features of the targeted concept, and highlighted common errors and misconceptions. A summary of the instructional designs of the two conditions can be found in *Table 2* below.

To ascertain the learning outcomes of the learning designs, three dependent variables were pursued – (1) procedural knowledge, (2) conceptual understanding, and (3) the ability to transfer – and these were measured using a post assessment measure administered immediately after the last instruction period. Students’ pre-requisite knowledge related to gradient was measured via a pre-test prior to the first period of implementation. The pre-test also served as a covariate, to control for any pre-existing differences in the pre-requisite knowledge of the topic between the two conditions. In addition to outcome measures, process measures in the form of the solutions generated during the problem-solving phase, video recordings of the lessons, and the field notes made by the research team, were also collected to shed light into the cognitive, intra- and interpersonal competencies that were exercised by the CLD students during the course of the intervention.

3. CLD learning materials

1) Problem task

A complex and open problem task that targeted the concept of gradient of linear graphs was used in the CLD condition’s problem-solving phase (see Ng et al., 2020, for the description of the problem task). Together with a team of experienced Singapore mathematics educators, an inquiry into the “gradient of

Table 2 *Instructional designs of CLD and DI conditions*

Phase sequence	Constructivist learning design (CLD)	Direct instruction (DI)
1	<p>Problem-solving</p> <p>Students worked collaboratively in dyads or triads to solve on a problem targeting a mathematical concept that they have yet to learn.</p> <p>The problem-solving activity took approximately 50 minutes to complete.</p>	<p>Instruction</p> <p>Teacher directly taught the targeted mathematical concept.</p>
2	<p>Instruction</p> <p>Teacher built on the student solutions generated in the <i>problem-solving</i> phase to instruct the targeted mathematical concept. Students applied newly gained knowledge via in-class and homework practices, targeting both the procedural knowledge and conceptual understanding required for the targeted concept. Additional practice questions were also introduced to supplement the ideas that were discussed during the consolidation of students’ ideas.</p>	<p>Problem-solving</p> <p>Students applied the newly gained knowledge via in-class and homework practices, targeting both the procedural knowledge and conceptual understanding required for the targeted concept.</p>

Note: The DI design appears to be less detailed than CLD, because the former is conventional and therefore the description of the tasks does not warrant additional explanation. The sequencing of the tasks in DI is also determined by the teacher and not the research team. Both the CLD and DI conditions do not significantly differ in the amount of practice. Rather, the main differences between CLD and DI lie in the sequencing of the problem-solving and instruction phases and practice as aligned to the demands of each design.

linear graphs” concept was conducted. The canonical gradient concept, which is a measure of steepness and direction of a straight line, is formulated as $\frac{\text{change in magnitude and direction of variable 1}}{\text{change in magnitude and direction of variable 2}}$ or $\frac{\text{Vertical change}}{\text{Horizontal change}}$ or $\frac{\text{Rise}}{\text{Run}}$. Four critical features that are associated to the gradient concept were identified: the (i) quantification/magnitude of steepness; (ii) the quantification of direction; (iii) the consideration of 2 dimensions/variables, the horizontal change and the vertical change; and (iv) the consideration of the ratio of the 2 variables. The problem task (see Figure 1) was designed to elicit these critical features, with varying slopes provided to allow students to model and study the gradient concept in terms of these features.

In the problem task, students were provided with a scenario of a person who was faced with 7 mountain trail sections of various steepness and different directions. Given the absolute height, horizontal distance, and slope length of each of the trail sections, students were instructed to use the information provided to develop as many mathematical measures as possible to help the person characterise the steepness and direction of each mountain trail section and to rank them accordingly. The values of the variables and the direction of the slopes were varied to elicit various conceptual features of gradient. For example, there are two trail sections that illustrate how slopes with the same horizontal distance or vertical height could

Figure 1 Problem task designed for the problem-solving phase of the CLD condition (“The Mountain Trail”), targeting the concept of gradient of linear graph

have different steepness (e.g., comparing trail sections CD and EF which have same horizontal distance but different steepness) and another pair of trail sections that illustrate how slopes with the same steepness could have different directions (e.g., comparing trail sections AB and GH).

2) Higher-order practice questions

To supplement the ideas that were discussed during the consolidation phase, three higher-order practice questions (see *Appendix Figure 1*) were given to students from the CLD class during the last lesson of the intervention. These questions are contextual questions that extend students' understanding of gradient of linear graphs as rate of change of one variable (e.g., the height of the water in the container) with respect to another (e.g., time).

4. Materials

1) Pre-test

A 7-item paper and pencil test was developed and administered to measure students' pre-requisite and prior formal knowledge for the study (see *Appendix Figure 2* in the Appendix for sample questions). To ensure both face and content validity, the pre-test was developed with a team of Singapore educators who had extensive experience in instructing mathematics at secondary level. Four of the questions, all multiple-choice questions (MCQ), examined students understanding of the concepts of lengths, ratios, coordinates, and angles, which are important to the understanding the concept of gradient. The remaining 3 questions, with 1 MCQ and 2 constructed-response questions, were designed to see if students knew the canonical gradient concept. The 4 pre-requisite items were employed as the covariate for the analysis of learning outcomes (see Results section) and shown to be fairly reliable ($\alpha=.65$).

2) Process measures

During the problem-solving phase, the CLD students were provided with pieces of blank papers to generate as many solutions as they can for the problem task (see *Figure 1*) and instructed to avoid erasing anything that they might have produced. The *student-generated solutions* were collected at the end of the lesson and later analysed by the researchers. Specifically, the following information was extracted and coded from the student artifacts: (1) the total number of solutions that each group produced, (2) the unique approaches that students used to solve the problem, (3) the variety or types of solutions that were generated, and (4) the differences between each type of solution in relation to the critical features of the gradient concept. These data were then used to inform us about CLD students' persistence, mathematical problem handling/modelling, inventiveness, and mathematical reasoning and thinking, respectively (see *Appendix Table 1*).

Two research assistants observed the problem-solving lesson and took down *field notes* with the

following guiding questions: (1) Did the students exhibit on-task or off-task behaviours in the classroom?; (2) Did the students communicate mathematical ideas with their teachers and/or peers?; and (3) On a scale of 1 to 4 (1 - *not engaged in any form of collaboration*, 4 - *highly collaborative with group members*), how would you rate the collaborative engagement of each student? In the instruction phase of both CLD and DI conditions, at least one research assistant was present. Field notes were taken with the following guiding questions: (1) What is the structure/flow of the lesson? (2) What did the teacher cover during the lesson? (3) How did the students react to the lesson, e.g., did they communicate mathematical ideas with their teacher or peers? In addition, all instructional lessons in both the CLD and DI lessons were *videotaped*. Both the field notes and video data were used to examine how teachers enacted their lessons and document students' use of communication and collaboration competencies as they engaged in deeper learning. Appendix Table 1 of the Appendix provides a summary of the process measures and the cognitive, intrapersonal, and interpersonal competencies that were examined.

3) Post-test

A 12-item post-test was used to measure students' knowledge on gradient of linear graphs after the intervention (see Appendix Figure 3 in the Appendix for sample questions). Like the pre-test, to ensure both face and content validity, the post-test was developed with a team of educators and education researchers who had extensive experience in instructing mathematics at secondary level. Among the items, 11 were MCQs and 1 was a constructed-response question. The post-test consists of four subscales, each marked out of 10 marks: (1) *procedural knowledge* (3 MCQ items), which assesses students' ability to calculate and compare the numerical values of gradients of linear graphs and interpret the gradient in a context; (2) *conceptual understanding* (3 MCQ items), which assesses students' knowledge on the meaning and mathematical properties of the gradient of linear graphs; (3) *ability to transfer to similar contexts (near transfer)*; 2 MCQ items), which ascertains students' ability to solve problems with both graphical and algebraic knowledge of gradient; and (4) *ability to transfer to different contexts (far transfer)*; 3 MCQ, 1 constructed response), which assesses students' ability to solve problems involving gradients with no given numerical values or unknowns, and to understand more advanced concepts like gradient of curves graphically. The post-test was administered in paper-and-pencil format. The full test was found to be fairly reliable in the present study (with Cronbach alphas at .60, with and without the constructed-response item).

5. Procedures

Prior to the study, the teacher who taught using the CLD approach underwent a 2-hour professional training session conducted by a member of the research project. The purpose of the training session was to familiarise the teacher with the learning design and the associated materials (e.g., problem task), as well

as the constructivist learning theory that underlies the learning design. Since DI is the dominant model of instruction that many Singapore teachers draw on (Kaur et al., 2019), the teacher in the DI condition was not given any training.

At the outset of the intervention, the pre-test was administered to both the CLD and DI classes. After which, the CLD class spent approximately 250 minutes of instructional time to complete the gradient of linear graphs unit. During the first lesson, students were given approximately 50 minutes to work on the complex, open-ended problem task in dyads and triads. They were instructed to generate as many possible solutions as possible and their teacher was present to facilitate the problem-solving session. The teacher then spent another 45 minutes on the second lesson to compare and contrast the student-generated solutions, evaluate their affordances and constraints, identify the relevant critical features of the concept, and assemble the ideas toward the canonical concept. The last three lessons were spent on regular classroom practices and the higher-order practice questions. In contrast, the DI class spent approximately 200 minutes of instructional time. The first lesson was used for the direct instruction of the gradient concept, and subsequent lessons was employed for classroom practices to reinforce what was taught in the first lesson. Since 50 minutes was devoted for the problem solving phase in the CLD, we could argue that both conditions have roughly the same amount of instructional and practice time. Upon completion of the unit, the post-test was administered to both classes.

IV. RESULTS

1. Process measures

CLD students' use of cognitive and intrapersonal competencies was captured by student-generated solutions, which were collected after the problem-solving lesson of the CLD class. Two research assistants were tasked to code the (1) total number of solutions that each group had produced, and (2) approaches that the students used in each solution. In cases where there were discrepancies, a third coder was called in to resolve any disagreements. Interrater reliability analysis was conducted, and it was found that both coders were in high agreement with one another ($Kappa=.90$).

1) Application of intrapersonal competencies

From the 10 groups of student solutions, an average of 4.00 solutions ($SD=1.94$ solutions) was produced, ranging from 1 to 8 solutions per group. While none of the students was able to produce the canonical gradient formula, the solution range provided evidence of students' *self-directedness* as they were able to work out at least 1 solution on their own without explicit guidance from the teacher. Only 1 out of the 10 groups produced a single solution, which indicated that the students had given up after a single attempt to the problem task. The remaining 9 groups produced two or more solutions, which are signs of *persistence*,

as they continued to put in effort to generate as many solutions as they could despite the impasse that they faced.

2) Application of cognitive competencies

In terms of mathematical problem handling and modelling competency, the various ways in which students had approached the problem were analysed. Across all 10 groups, we identified a total of 14 distinct approaches that students used to solve the problem. Three of the approaches utilized existing parameters in the problem task (e.g., comparing the horizontal distances between slopes), three involved the creation of new parameters that were not given in the problem task (e.g., change in height, angles from a certain reference point), and eight performed some forms of manipulation on the parameters to generate new measures of steepness and direction (e.g., subtracting horizontal distance from slope length). The different approaches demonstrate students' competent use of available data in a contextual problem to develop mathematical models or measure of a phenomena, i.e., gradient of slope.

While the sheer number of solutions or approaches could indicate students' level of persistence, this is insufficient to indicate students' *inventiveness*. This is because students could generate multiple solutions of the same nature without variety. As such, we analysed how solutions relate to or differentiate from one another and classified them accordingly. Based on our analysis, the solutions that the students produced conformed to four categories that were related to the critical features: (1) the use one variable/dimension (e.g., consider only height, horizontal distance, or slope length in the measure of slope), (2) the use of a combination of two variables/dimensions (e.g., taking the difference between horizontal and slope lengths), (3) a ratio of two variables/dimension without considering the direction (e.g., taking the ratio of slope length and the absolute change in height), and (4) the use of angles. *Table 3* below shows examples of the solution types.

Among the 10 groups, two groups produced solutions of 1 solution type, while the rest had a mixture of 2 to 4 solution types in their work. From the solutions, it was apparent that students had engaged in *convergent thinking* by activating and applying their prior knowledge (e.g., lengths, ratio, angles) when making sense of the problem situation, and engaged in divergent thinking by creating new measures through combinations of their formal knowledge (e.g., combining idea of ratio and lengths). Collectively, the solutions demonstrate students' inventive thinking capacities via their creative use of prior knowledge in designing new measures of steepness and direction of a slope.

Finally, to examine if CLD students had engaged in mathematical thinking and reasoning during the problem-solving activity, we analysed how they moved from one solution to the next and uncovered their thinking and reasoning behind their change in approach. To illustrate, *Figure 2* shows the progression of the solutions that was produced by one of the groups in the CLD class. The group began by using a change in height to rank the steepness and direction of each slope. Upon realizing that the ranking did not coincide with their qualitative observations, they moved on to subtract slope length from horizontal

Table 3 Solution types for the “Mountain Trail” problem

Type	Description	Example
1	Solutions that consider only one dimension/ variable when addressing either steepness or direction of the trail sections	<p> $A \& B \rightarrow 400m - 0$ 3rd $= 400m$ $B \& C \rightarrow 0m$ 5th $C \& D \rightarrow 400m$ 3rd $D \& E \rightarrow 200m$ 4th $E \& F \rightarrow 600m$ 2nd $F \& G \rightarrow 800m$ 1st $G \& H \rightarrow 400m$ 3rd </p> <p>Determining the steepness and direction of the slope by subtracting slope length (first variable) from horizontal distance (second variable)</p>
2	Solutions that consider a combination of two dimensions/variables when addressing both steepness and direction (where applicable) of the trail sections	<p> $A-B \Rightarrow 720m - 600m = 120m$ $B-C \Rightarrow 200m$ $C-D \Rightarrow 570m - 400m = 170m$ $D-E \Rightarrow 360m - 300m = 60m$ $E-F \Rightarrow 720m - 400m = 320m$ $F-G \Rightarrow 1140m - 800m = 340m$ $G-H \Rightarrow 720m - 600m = 120m$ </p> <p> $F-G$: Rank 1 $E-F$: Rank 2 $B-C$: Rank 3 $C-D$: Rank 4 $A-B$ and $G-H$: Rank 5 $D-E$: Rank 6 </p> <p>this is to show the difference of the steepness with the so called flat land, so the higher the amount, the steeper the mountain is</p> <p>Determining the steepness and direction of the slope by finding the ratio between slope length and the absolute change in heights</p>
3	Solutions that consider the ratio of two dimensions/variables when addressing the steepness of the trail sections, without considering the direction	<p> $A \rightarrow B = 1.8$ $B \rightarrow C = 0$ $C \rightarrow D = 1.425$ $D \rightarrow E = 1.8$ $F \rightarrow E = 1.2$ $F \rightarrow G = 1.425$ $G \rightarrow H = 1.8$ </p> <p> 1: $E \rightarrow F$ 2: $C \rightarrow D$ 3: $F \rightarrow G$ 4: $D \rightarrow E$ 5: $G \rightarrow H$ 6: $A \rightarrow B$ 7: $B \rightarrow C$ </p> <p>Determining the steepness and direction of the slope by finding the ratio between slope length and the absolute change in heights</p> <p>Method $\rightarrow$ The slope length divide by the height</p>
4	The consideration of angles when addressing the steepness and direction (where applicable) of the trail sections	<p> Steepest Descending $A-B = 34^\circ$ $B-C = 0^\circ$ $C-D = 35^\circ$ $D-E = 56^\circ$ $E-F = 57^\circ$ $F-G = 79^\circ$ $G-H = 33^\circ$ </p> <p>Determining the steepness and direction of the slope with angles</p>

Note: As these are authentic student solutions, calculation errors are expected.

Figure 2 Progression in solutions for the “The Mountain Trail” task for one group

distance. The progression in sophistication of solutions demonstrated students' engagement in critical and reflective thinking, as they evaluated the viability of their solutions (i.e., whether their measures of steepness and direction reflect the reality that they observed) and sought to improve their measures by using more sophisticated approaches like angles and ratio of slope.

3) Interpersonal competencies observed

Two research assistants, who were present during the problem-solving phase of the CLD class, compared their field notes and agreed that the CLD students had worked collaboratively to generate as many solutions as they could for the “gradient of linear graphs” problem task. On a scale of 1 to 4 (1 – not engaged in any form of collaboration, 4 – highly collaborative with group members), all students were given a rating of either a 3 or 4, except for one student who was rated with a 1 by both research assistants as she appeared to be disengaged from the whole activity. Based on the field notes, CLD students were observed to be actively engaged in mathematical discussions, expressing their personal perspectives of the problem to their group members, and critiquing and building on each other's ideas. Faced with the complex data that could afford multiple solutions, groups were observed to employ different strategies in

solution generation. In some groups, members first worked individually before bringing the solutions to the table for discussion. In others, members started with a discussion of possible ideas to solve the problem before the assignment of different methods to each member. Based on observations, the CLD students appeared to have utilised their collaboration and communication skills during the problem-solving session.

The video analysis indicated that the CLD afforded the teacher and students more opportunities to engage in mathematical discourse during the instruction phase. For example, to follow-up with the activity in the problem-solving phase, the CLD teacher began his instruction by asking his students to express their informal understanding of steepness and building upon their intuitive notions to get them engaged in the gradient concept:

T: What is steepness to you after doing the activity? ...

S: So, steepness to me is like the difficulty ... the amount of difficulty put in when you want to walk up that slope.

T: Very good ... so steepness to you is ... how difficult it is to climb a slope, is it? Like when you climb a hill, definitely the slopes are of different steepness, some slopes are easy to climb, some slopes are not easy to climb ...

We also observed considerably more student-teacher interactions or dialogue in the CLD instruction lessons as the class discussed the affordances and constraints of the student-generated solutions. The transcript below, for instance, illustrates the student-teacher interaction that was captured when the class discussed whether using slope length alone is a good measure of gradient:

T: [after showing how some students used slope length as a measure of steepness and direction] So, is this a good measure?

Ss: No.

T: Why not? ... Why is it not a good measurement?

S: Because AB and EF are not the same ...

T: So, because AB and EF have the same slope length, but are they the same steepness?

S: No.

T: So, slope length is not a good comparison.

In another instance, we saw how the CLD class engaged in a whole class discussion when they talked about whether slope CD (gradient=1.00) and slope FG (gradient=-1.00) have the same steepness:

T: [when ranking the steepness of slopes in the problem task after calculating the gradient] which one is the next steepest (after CD)?

S: FG

T: Oh, ok, why do you say it's FG?

Ss: [from other students] no, it's wrong. It should be BC (gradient=0.67)

T: Wait ... you were saying FG?

S: Teacher, does the negative (sign of the gradient value) matter?

T: Does the negative matter?

Ss: [some said 'no', some said 'yes']

During this lesson episode, there was a rich exchange of students' opinions, which led the teacher to bring out the idea of steepness and direction. Evident from the transcript, while slope CD and FG has different direction, the absolute value of their gradients suggest that they hold the same steepness.

In contrast, the DI teacher's introduction of the gradient concept was more teacher-led and did not delve into students' notions. The DI teacher began his instruction of gradient by asking the class to guess where the steepest slope could be found in the world. After introducing that slope, he noted that when researching how to describe the steepness of the road, a certain number was given to him. He explained that the number was a ratio of the height of the slope with respect to the horizontal ground. He gave another example, this time of a canal with a gentler slope and its associated height-distance ratio. He noted that the numbers derived from the ratios are known as gradient.

T: Now, I think from the two examples shown, I think you should roughly know what's the meaning of gradient. We are trying to find out how steep something is. For the first example, we are trying to find out how steep something is. For the second example, we are trying to find out steep is a canal... cause you all can understand that the canal is not flat.

He then directed the students to the textbook, and from the relevant pages, started showing that the gradient was expressed as the ratio between the vertical height and horizontal length of a slope, or rise over run. In line with most direct instruction classes, lessons were expository in nature, with the teacher leading the introduction of concept, and showing how the concept was formulated and applied to problems. Student talk was mainly for clarification of whether the concept was correctly applied or their answers to practice questions were answered correctly.

Overall, the CLD students exercised their communication and collaboration skills when solving the complex problem during the problem-solving phase and when discussing the canonical idea of gradient in the instruction phase.

Table 4 Descriptive and Inferential statistics of control and outcome measures

Facet	# items	Max score	Conditions						
			CLD			DI			
			n	M	SD	n	M	SD	
Control measure									
Pre-requisite knowledge	4	10	23	4.78	2.71	18	5.56	3.04	n.s.
Outcome measures									
Procedural knowledge	3	10	23	7.68	2.12	18	6.48	2.91	n.s.
Conceptual understanding	4	10	23	5.94	3.89	18	1.86	2.05	F(1, 38) = 16.01, p<.001
Near transfer	2	10	23	3.91	3.00	18	1.94	3.04	F(1, 38) = 5.42, p<.05
Far transfer	4	10	23	2.83	2.39	18	1.53	1.74	F(1, 38) = 4.85, p<.05

2. Outcome measures

An ANOVA that was conducted on the 4-item pre-test that assesses pre-requisite knowledge (see Table 4 for the descriptive statistics) revealed no significant difference in both conditions. In addition, analysis of the 3 items in the pre-test that directly interrogated whether students knew the gradient formula revealed that both conditions had no knowledge of the formula.

Controlling for the effects of students' pre-requisite knowledge using 4-item pre-test, a Multivariate Analysis of Covariance (MANCOVA) was conducted to examine if students in the CLD and DI class differ in the four outcome measures. The results showed that two classes were significantly different in the learning outcomes, $F(4, 35) = 5.95, p < .01$; Wilk's $\Lambda = .60$, partial $\eta^2 = .41$. Subsequent tests of between-subject effects further indicated that the CLD class had significantly higher scores for conceptual understanding ($F(1, 38) = 16.01, p < 0.001$), near transfer ($F(1, 38) = 5.42, p < 0.05$), and far transfer ($F(1, 38) = 4.85, p < 0.05$) items. There was no significant difference between the two classes for the procedural knowledge items.

V. DISCUSSION AND CONCLUSION

To prepare students for the 4IR in mathematics classrooms, our study proposes a learning design that affords students to engage in deeper learning and develop transferable skills and knowledge (Pellegrino & Hilton, 2012). Informed by constructivist principles of learning, the learning design, CLD, adopts a "problem-solving first, instruction later" approach (Loibl et al., 2017). While past research has indicated the potential of similar designs in achieving deeper learning (e.g., Sergis & Sampson, 2019), many of these designs did not address all domains in deeper learning. Building on these gaps, our study showed that when students were given the opportunity to work collaboratively on a complex, open-ended problem that targets a mathematics concept that had not been taught formally, and then taught the targeted concept using the solutions that they generated, student were able to apply their cognitive, intrapersonal,

and interpersonal competencies and engage in deeper learning, which in turn conferred learning benefits. Post assessments ascertaining the efficacy of CLD revealed that CLD students not only outperformed their DI counterparts in items that assessed conceptual understanding of the targeted mathematics concept, but also on items that examined their ability to transfer what they learnt to more novel settings, whether the contexts were similar (i.e., near transfer situations), or contexts that required advanced knowledge of gradient in unfamiliar contexts (i.e., far transfer situations). These findings provide a positive indication that the CLD has engendered deeper learning processes to afford the cultivation of transferrable skills and knowledge.

The lesson observations and the analysis of the process measures (e.g., student-generated solutions) provided insights into how deeper learning could be enacted in the mathematics classrooms, and how CLD could afford students to experience uncertainties in the learning process. During the problem-solving lesson, CLD students had to navigate through a problem with an unknown solution, akin to the uncertain and challenging 4IR environments that await them in the future. Working within their resources, students harnessed their mathematical problem handling skills, engaged in micro-modelling (Watson & Mason, 2006) with the data presented, and exhibited qualities of persistence, self-directedness, and inventiveness as they tried to generate as many solutions as they could without explicit guidance from their teacher. Working in groups to navigate their way to solve the problem, students honed the necessary interpersonal competencies of communicating and collaborating with others while negotiating the meaning of their solutions. The student-generated solutions illustrated how students had engaged in mathematical reasoning and thinking, as they sought to refine their mathematical ideas and progress through solutions of different levels of sophistication. These processes provided students the opportunities to apply various cognitive, intrapersonal, and interpersonal competencies (e.g., inventiveness, persistence, self-directedness) that are necessary for successful transfer. Such experience would in turn prepare them for future novel and unfamiliar problem situations inside and outside of school.

Despite the challenging problem-solving phase, it seems that CLD also provided the room for learner agency and self-regulation. This was possible with the affective support that the teacher provided during the problem-solving phase and the facilitatory stance adopted by the teacher as he built upon students' solutions in the instruction phase. The teacher played the role of a negotiator rather than an authority, as students were engaged and persuaded to see that their solutions had affordances and constraints depending on the problem context (diSessa & Sherin, 2000; Kapur & Bielaczyc, 2012). These factors possibly contributed to learners' willingness to persist at the challenging task, as the affective support provided by the teacher has helped to offset any negative feelings towards the uncertainty in the learning environment.

The CLD learning environment is aligned with various pedagogical techniques such as problem-based, co-operative, and experiential learning that are recommended by alternative 21st century competences frameworks, such as the "Partnership for 21st Century Skills" and "European Reference Frameworks" (Voogt

& Roblin, 2012). However, what the CLD demonstrates are the possible mechanisms that one can design for to effect knowledge construction that can achieve the deeper learning process. In line with many “problem-solving first, instruction later” designs (Loibl et al., 2017) such as Productive Failure (Kapur, 2008, 2010; Kapur & Bielaczyc, 2012) and the Japanese Open-Ended Approach (Becker & Shimada, 1997), the problem was engineered in ways that not only afford the exploration and generation of many solutions, but also allowing for students to experience impasse if they relied on tried and tested methods. This feature of CLD is also a window to students’ prior knowledge and intuitive conceptions, which is an important resource for teachers to build upon, in examining their viability vis-à-vis the targeted concept. Similar to the principles for the consolidation phase in the Productive Failure framework (Kapur & Bielaczyc, 2012) the teacher activates and describes the various solutions, and through a whole class discussion, compares and contrasts the affordances and constraints for these solutions, getting students to notice the critical features of the targeted concept. The explanation and elaboration of these features should further aid students’ encoding process, allowing them to develop a deep understanding of the concept. Once all the critical features were being attended to, teachers further explain how these critical features were assembled in the canonical concept. Such a classroom discourse affords students to understand the targeted concept not only in connection to their prior understanding, but also with its multiple and varied representations. These would in turn allow them to develop a deep, connected, and organised system of knowledge which would allow for flexible understanding, and support transfer (Goldman & Pellegrino, 2015).

The tapping of prior knowledge resources in the learning of a new concept echoes the process of individual unlearning, learning, and re-learning amidst the ever-changing landscape of the future workplace in management literature (Hislop et al., 2014; Perschl, 2019; Sharma & Lenka, 2019). The ability to learn is what differentiates workers in the 4IR environment (Ra et al., 2019), but how this learnability process takes shape, particularly how people deal with obsolete knowledge to embrace new knowledge, is still contentious and under researched (Hislop et al., 2014; Perschl, 2019; Sharma & Lenka, 2019). The dynamics between prior and new knowledge in the CLD could possibly shed light into this learnability process. Gaining the insight into how prior and new knowledge were connected allowed CLD to better navigate newer environments (e.g., transfer questions in the post test) compared to their DI counterparts. In DI environments, where the teacher sets the stage of how new knowledge is to be organised, students might not know the significance of their prior knowledge, and hence are unaware how they are unlearning old concepts. By getting students immersed in the CLD, this may not only get them to achieve a deeper and more connected knowledge, it might also help them develop the important skills to unlearn and relearn in the demanding 4IR environment.

1. Implications for practice

While CLD seems promising in preparing students for the 4IR environment, sustaining such an approach in the classroom is dependent on teacher capacity (Ball et al., 2008; Shulman, 1986). Teachers play a critical role in determining the degree of success in implementing instructional innovations (Doyle & Ponder, 1977; Ghaith & Yaghi, 1997; Guskey, 1988; Kennedy & Kennedy, 1996; Stein & Wang, 1988; Zhao et al., 2002). As curriculum developers attempt to bring transformative change in learning and teaching, they face major challenges of teachers' reluctance to adopt new practices (Lortie, 1975) and of their continued use of the innovation in ways that are congruent with their intent (Fishman et al., 2011). Professionally supporting teachers in the development of the necessary content knowledge and pedagogical content knowledge (Shulman, 1986; Ball et al., 2008) is therefore important, as we have done for the study. In addition, adequate support from the school management, the willingness of teachers to set aside time and effort to attempt a new pedagogy, and the building of professional learning communities are also important to help sustain the practice.

2. Limitations of study

With the CLD, we sought to help students engage in the process of deeper learning and exercise their cognitive, intrapersonal, and interpersonal competencies in the mathematics classroom. While our findings suggest that CLD seems effective in developing students' conceptual understanding in mathematics and ability to engage in cognitive transfer, it is important to note that the present study was conducted over a short intervention period within a particular mathematics unit (i.e., gradient of linear graphs). As such, we are unable to ascertain (1) the generalisability of the effectiveness of CLD in other mathematics units, and (2) the long-term effects of the intervention (e.g., whether the current intervention affected students' subsequent learning), and (3) whether repeated use of CLD could help to develop intrapersonal, and interpersonal competencies in the long run.

In addition, we acknowledge that CLD's learning outcomes are more aligned with cognitive constructivist principles and did not deeply consider, operationalise, and analyse interpersonal and intrapersonal skills. Moreover, what remains unclear is whether the intrapersonal and interpersonal competencies that were used in the CLD are indeed transferrable. Different contexts require different interpersonal skills, and it is possible that the nature of communication adopted during face to-face open ended tasks may differ from those in the technologically powered 4IR environment. Current instruments to assess these intrapersonal and interpersonal skills are limited (Pellegrino & Hilton, 2012), and we await future research to address the possibilities of deducing the transferability of these soft competencies. As the problem-solving phase of the CLD provides opportunities for teachers and researchers to capture the intrapersonal and interpersonal skills of the students (e.g., through surveys, analysis of students' audio discussions), we suggest that future

research could conduct CLD across multiple mathematics units to a same group of students and track the development of their competencies through the various iterations of problem-solving.

There are currently very few studies that examined how the competencies necessary for students to prepare for the 4IR could be operationalised and integrated into a learning design that can be leveraged in current instruction. The CLD that we propose present a possibility. In addition, much research commented on how learning needs to be transformed in higher education (e.g., Gleason, 2018), but our study suggests that such transformations can be realised and nurtured in the younger secondary populations. The study shows that that mathematics instruction should emphasise more on the processes of problem solving to afford deeper and meaningful learning and the development mathematical habits and dispositions in students, and these would have benefits in understanding and transfer. Such emphases may be more sustainable in preparing students for a demanding and uncertain future.

CONFLICTS OF INTEREST

No potential conflict of interest relevant to this article was reported.

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